DOCUMENT RESUME

ED 120 001 95 SE 020 462

AUTHOR	Higgins, Jon L., Ed.
TITLE	Research Reporting Sections, Annual Meeting of National Council of Teachers of Mathematics (54th, Atlanta, Georgia, April 21-24, 1976).
INSTITUTION	ERIC Information Analysis Center for Science, Mathematics, and Environmental Education, Columbus, Ohio.; Ohio State Univ., Columbus. Center for Science and Mathematics Education.
SPONS AGENCY	Office of Education (DHEW), Washington, D.C.
PUB DATE	Apr 76
NOTE	117p.
AVAILABLE FROM	Information Reference Center (ERIC/IRC), The Ohio State University, 1200 Chambers Road, 3rd Floor, Columbus, Ohio 43212 ($3.00)
EDRS PRICE	MF-$0.83 HC-$6.01 Plus Postage
DESCRIPTORS	Algorithms; Attitudes; Evaluation; *Instruction; *Learning; Logic; Mathematical Concepts; *Mathematics Education; Problem Solving; *Research; *Teacher Education; Teaching Methods
IDENTIFIERS	*National Council of Teachers of Mathematics; NCTM; Research Reports

ABSTRACT

Abstracts of 28 research reports are provided. The reports were prepared by investigators for presentation at the 54th annual meeting of the National Council of Teachers of Mathematics. A broad range of topics related to mathematics education are covered. Three reports concern the effects of differing presentations of mathematics, four are related to attitudes of teachers or students, and three to interactions of personality variables with mathematics learning. Three papers concern problem solving, three deal with logic, and two concern the division algorithm. Other papers concern readability of achievement tests, learning of fractions, measurement, cultural differences in mathematics learning, uses of computer graphics, comparison of calculus courses in high school and college, effectiveness of a remediation program, the use of homework, and field testing of a teacher education program. (SD)

****
* Documents acquired by ERIC include many informal unpublished *
* materials not available from other sources. ERIC makes every effort *
* to obtain the best copy available. Nevertheless, items of marginal *
* reproducibility are often encountered and this affects the quality *
* of the microfiche and hardcopy reproductions ERIC makes available *
* via the ERIC Document Reproduction Service (EDRS). EDRS is not *
* responsible for the quality of the original document. Reproductions *
* supplied by EDRS are the best that can be made from the original. *
****

U.S. DEPARTMENT OF HEALTH,
EDUCATION & WELFARE
NATIONAL INSTITUTE OF
EDUCATION
THIS DOCUMENT HAS BEEN REPRODUCED EXACTLY AS RECEIVED FROM THE PERSON OR ORGANIZATION ORIGINATING IT. POINTS OF VIEW OR OPINIONS STATED DO NOT NECESSARILY REPRESENT OFFICIAL NATIONAL INSTITUTE OF EDUCATION POSITION OR POLICY

MATHEMATICS EDUCATION INFORMATION REPORT

MATH
EDUC
INFO
REPO

ED120001

RESEARCH REPORTING SECTIONS
NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS
54th ANNUAL MEETING

Atlanta, Georgia
April 21-24, 1976

THE ERIC SCIENCE, MATHEMATICS AND ENVIRONMENTAL EDUCATION CLEARINGHOUSE
in cooperation with
Center for Science and Mathematics Education
The Ohio State University

SE 020 462

MATHEMATICS EDUCATION REPORTS

RESEARCH REPORTING SECTIONS
NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS
54th ANNUAL MEETING

Atlanta, Georgia
April 21-24, 1976

ERIC Information Analysis Center for
Science, Mathematics, and Environmental Education
1200 Chambers Road
The Ohio State University
Columbus, Ohio 43212

PREFACE

The ERIC Information Analysis Center for Science, Mathematics, and Environmental Education has compiled abstracts of the research papers to be presented at the 54th annual meeting of the National Council of Teachers of Mathematics. Selection of papers was made by Professor L. Ray Carry of The University of Texas at Austin and a committee of the AERA Special Interest Group for Research in Mathematics Education. Minor editing has been done by the ERIC staff to provide a general format for the papers. Many of the papers that are abstracted here will be published in full in journals or will be made available through the ERIC system. These will be announced in Research in Education and in the journal Investigations in Mathematics Education.

December, 1975

Jon L. Higgins
Associate Director for
Mathematics Education

Sponsored by the Educational Resources Information Center of the United State Office of Education and the Center for Science and Mathematics Education, The Ohio State University.

This publication was prepared pursuant to a contract with the Office of Education, United States Department of Health, Education, and Welfare. Contractors undertaking such projects under Government sponsorship are encouraged to express freely their judgment in professional and technical matters. Points of view of opinions do not, therefore, necessarily represent official Office of Education position or policy.

CONTENTS

NCTM RESEARCH REPORTING SECTIONS
1976 Annual Meeting

Morton Friedman
TEACHER QUESTIONS AND PUPIL PERFORMANCE 2

William A. Juraschek
THE INTERPRETATION OF THE CONNECTIVE OR IN DISJUNCTIVE ARGUMENTS 8

Suzanne K. Damarin
INTERPRETATIONS OF LOGICAL CONNECTIVES BY PRE-SERVICE ELEMENTARY SCHOOL TEACHERS 14

Thomas R. Post; William H. Ward, Jr.; and Victor L. Willson
DIFFERENCES BETWEEN TEACHERS' ATTITUDINAL SELF-RATING AND THEIR PRINCIPALS' TEACHER IDEAL ON A MATHEMATICS INVENTORY 18

Betsey S. Whitman
INTUITIVE EQUATION SOLVING SKILLS AND THE EFFECTS ON THEM OF FORMAL TECHNIQUES OF EQUATION SOLVING 28

Lionel Pereira-Mendoza
THE EFFECT OF INSTRUCTION IN HEURISTICS ON THE ABILITY TO SOLVE OPEN ENDED MATHEMATICAL PROBLEMS 32

Douglas G. Garbe
CONCEPTS FOR MATHEMATICS TERMS COMPARED BETWEEN INDIAN STUDENTS AND THEIR NON-INDIAN PEERS 36

Charles D. Friesen
THE EFFECT OF EXPLORATORY AND REVIEW HOMEWORK EXERCISES UPON ACHIEVEMENT, RETENTION, AND ATTITUDE IN A FIRST-YEAR ALGEBRA COURSE 39

Charles E. Lamb
A STUDY OF THE YOUNG CHILD'S ABILITY TO USE PROPERTIES OF THE MEASUREMENT FUNCTION 44

Linnea Weiland and Robert Madell
A DESCRIPTION OF THE DIFFICULTIES SELECTED EIGHT YEAR OLD CHILDREN HAVE IN LEARNING THE DISTRIBUTIVE ALGORITHM FOR DIVISION 46

Gilberto Jose Cuevas
THE EFFECTS OF PRECISE THINKING ALGORITHMIC EXERCISES ON THE FIFTH GRADE STUDENT'S ABILITY TO PERFORM THE DIVISION ALGORITHM 49

Thomas A. Egan; John J. Kerrigan; Mark F. Wiener and C. Ralph Verno
IMPLEMENTATION OF AN INDIVIDUALIZED APPROACH WITH COLLEGE FRESHMAN MATHEMATICS SURVEY COURSES: A STUDY 51

Dr. Joe Dan Austin and Dr. Mary Ann Byrne
READABILITY OF ACHIEVEMENT TESTS 56

Michael Wheat and Gerald Kulm
THE EFFECTS OF VOCABULARY AND TRANSLATION TRAINING ON SOLVING VERBAL PROBLEMS 59

Dorothea Dora Smith
EFFECTIVENESS OF AN INNOVATIVE CURRICULUM IN MATHEMATICS ON ATTITUDE TOWARD MATHEMATICS, SELF-CONCEPT, AND ACHIEVEMENT IN MATHEMATICS 62

Robert R. O'Reilly
ATTITUDES CONCERNING THE NATURE OF MATHEMATICS OF SECONDARY SCHOOL STUDENTS 65

James D. Gates
CONFERENCES OF THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS: PERCEPTIONS OF SELECTED PERSONNEL 71

George Aman
AN ANALYSIS OF THE RELATIVE EFFECTIVENESS OF VARIOUS REMEDIAL MATHEMATICS PROGRAMS FINANCED BY TITLE I, ESEA FUNDS IN NEW YORK STATE 74

Margaret A. Farrell; Walter A. Farmer and Richard Clark
FIELD TESTING A SYSTEMS ANALYSIS MODEL FOR MATHEMATICS/ SCIENCE TEACHER EDUCATION 76

Bernard Baltz
FUTURE AND PRESENT EFFECTIVENESS OF COMPUTER GRAPHICS 79

Dennis M. Luciano and Agnes M. Rash
THE EFFECTS OF CALCULUS IN HIGH SCHOOL ON COLLEGE CALCULUS PERFORMANCE 84

Daryle Cline May
AN INVESTIGATION OF THE RELATIONSHIP BETWEEN SELECTED PERSONALITY CHARACTERISTICS OF EIGHTH-GRADE STUDENTS AND THEIR ACHIEVEMENT IN MATHEMATICS 87

Carol F. Novillis
THE EFFECT OF THE LENGTH OF THE NUMBER LINE AND THE NUMBER OF LINE SEGMENTS IN EACH UNIT SEGMENT ON SEVENTH-GRADE STUDENTS' ABILITY TO LOCATE PROPER FRACTIONS ON THE NUMBER LINE 89

Lyle R. Smith and James H. Rollins
ASPECTS OF TEACHER DISCOURSE AND STUDENT LEARNING OF A MATHEMATICAL CONCEPT 92

Thomas P. Carpenter; Douglas B. McLeod and Romualdas Skvarcius
COGNITIVE STYLE AND MATHEMATICS LEARNING: THE INTERACTION OF FIELD INDEPENDENCE AND INSTRUCTIONAL TREATMENT IN NUMERATION SYSTEMS 98

William J. O'Donnell
EFFECTS OF STUDENT AND TEACHER COGNITIVE PREFERENCES ON JUNIOR HIGH MATHEMATICS ACHIEVEMENT 101

Mary K. Abkemeier and Fred H. Bell
RELATIONSHIPS BETWEEN VARIABLES IN LEARNING AND MODES OF PRESENTING MATHEMATICS CONCEPTS 104

Lars C. Jansson
A COMPARISON OF TWO APPROACHES TO THE ASSESSMENT OF DEDUCTIVE REASONING ABILITIES 108

RESEARCH REPORTING SECTION #1

Presider: L. RAY CARRY, The University of Texas at Austin
Austin, Texas

Reports: TEACHER QUESTIONS AND PUPIL PERFORMANCE

MORTON FRIEDMAN, Baruch College of CUNY
Brooklyn, New York

THE INTERPRETATION OF THE CONNECTIVE OR IN DISJUNCTIVE ARGUMENTS

WILLIAM A. JURASCHEK, The University of Texas at Austin
Austin, Texas

INTERPRETATIONS OF LOGICAL CONNECTIVES BY PRE-SERVICE ELEMENTARY SCHOOL TEACHERS

SUZANNE K. DAMARIN, The Ohio State University
Columbus, Ohio

DIFFERENCES BETWEEN TEACHERS' ATTITUDINAL SELF-RATINGS AND THEIR PRINCIPALS' TEACHER IDEAL ON A MATHEMATICS INVENTORY

THOMAS R. POST, University of Minnesota
Minneapolis, Minnesota

WILLIAM H. WARD, JR., University of Minnesota
Minneapolis, Minnesota

VICTOR L. WILLSON, University of Minnesota
Minneapolis, Minnesota

TEACHER QUESTIONS AND PUPIL PERFORMANCE

Morton Friedman
Baruch College of
The City University of New York

Purpose

The publication of Bloom's taxonomy (1) stimulated the interest of teacher-trainers and educational researchers in the questioning practices of classroom teachers. Several investigators used the Bloom taxonomy to report that the cognitive emphasis of teachers' classroom questions was at the lowest level. This finding was used to support the position that teachers can improve their instruction by emphasizing higher levels of questions. However, as Dunkin and Biddle (3) point out, the validity of this position is premised on two further assumptions: (a) teachers can be trained to vary the cognitive level of their questions and (b) pupil achievement is positively related to questioning at higher levels. Several experimenters (3, 4, 7) have used the Bloom taxonomy to train teachers to ask questions at higher levels. However, only two investigators have used the taxonomy to investigate the process-product relationship. Rogers (7) reported that the achievement of pupils whose teachers emphasized higher levels of questions in the context of elementary school social studies instruction did not significantly differ from the achievement of pupils whose teachers emphasized lower-level questions. Ragosta (8) reported that pupil growth was positively associated with teacher emphasis of lower-level categories and negatively associated with teacher emphasis of higher-level categories in first-grade urban classrooms.

The paucity of studies involving the relationship between teacher questioning and pupil achievement prompted the present study. Its purpose was to determine the relationship between teacher emphasis of a cognitive level and pupil achievement at that level. The context for the investigation was the high school geometry classroom.

Procedure

Subjects: Eight teachers participated in the study (on a voluntary basis). They were located in six different high schools (two pairs of teachers taught at the same school). Their teaching experience ranged from 3 to 25 years, with a median of 6 years. Four of the eight had earned a Master's degree in addition to the baccalaureate.

The schools were situated in middle-class communities in or near New York City. The classes were of "average ability" (based on previous teacher evaluation). The class size ranged from 24 to 35 with a median of 29.

Instruments: A modified version of the Bloom taxonomy was used to determine a teacher's emphasis of a cognitive level. This category system places a teacher's oral question in one of four categories: memory, comprehension, application, or higher-level. For a detailed description of the system see (6).

The investigator developed an objective test for the study to serve as the criterion measure of pupil achievement. The test measured achievement of geometric concepts at the four cognitive levels, memory, comprehension, application, and higher-level.

Several forms of a preliminary version of the examination were administered to a pilot group. Items were written in either the multiple-choice or open-ended format. This procedure enabled the investigator to improve the construction of the test with respect to clarity and attractiveness of incorrect multiple-choice options.

A three-judge panel of experts validated the final version of the examination for content and classification of cognitive level. It consisted of three memory items, five comprehension items, four application items, and six higher-level items.

Context: Each teacher taught two lessons. In each lesson the teacher introduced and proved a geometry theorem. The two theorems were the same for all teachers. They were the following: (a) The sum of the measures of the angles of a triangle is 180; (b) If the hypotenuse and a leg of one right triangle are congruent to the corresponding parts of a second right triangle, the triangles are congruent.

The investigator described the study to each teacher in general terms prior to the start. The teachers knew that teacher behavior was under investigation. They did not know that the investigation was concerned with the cognitive level of their questions.

Each teacher tape-recorded (audio) his lessons. He administered the examination during the period of instruction that followed the second lesson. All examinations were given at approximately the same time.

Analysis

The teachers' questions were coded by the investigator directly from the tapes using the category system described above. Inter-rater reliability (3) was determined to be 94 percent.

A teacher was designated as a member of the group that emphasized a cognitive level (E-group) if on both occasions he asked more than the median number of questions asked at that level. Similarly, if a teacher asked less than the median number of questions asked a particular level, he was designated as a member of the group that deemphasized the cognitive level (D-group).

Pupil achievement scores on each cognitive level of the test were calculated by determining the ratio of correct responses to total responses (p). A test of proportions (5) was used to compare the p-scores of the two groups.

Two p-scores were calculated for each group at each cognitive level. One score represented pupil achievement at that cognitive level and the other score represented pupil achievement on the remaining levels of the examination. The purpose of the second score was to consider the possibility of differences due to pupil ability rather than teacher behavior.

Results

The number of questions asked at each cognitive level by each teacher during each lesson is shown in Table 1. The data indicate that the median number of higher-level questions asked was zero and that no teacher asked higher-level questions during both lessons. Consequently, the comparison of p-scores is limited to the memory, comprehension, and application levels.

Table 1

Number of Questions Asked at Each Cognitive Level by Each Teacher During Each Lesson

Teacher	Lesson 1				Lesson 2			
	M[a]	C	A	H	M	C	A	H
1	12	26	13	0	1	2	5	0
2	4	15	10	1	10	7	5	0
3	5	39	24	0	10	35	13	0
4	13	30	9	0	10	31	3	0
5	13	44	8	0	21	31	5	0
6	12	38	4	0	23	41	4	0
7	9	39	6	1	17	58	19	0
8	5	29	5	0	7	16	7	1
Median	10.5	34	8.5	0	10	3[illegible]	5	0

[a] Capital letters represent cognitive levels. M denotes memory, C denotes comprehension, A denotes application, and H denotes higher-level.

For the memory level, as shown in Table 2, the p-score of pupils taught by the E-group was significantly higher (at the .01 level) for the memory test items than the p-scores of pupils taught by the D-group. However, there was no significant difference between the p-scores of the two groups for the remaining test items.

Table 2

Test of Proportions Comparing Achievement on the Criterion Test of Pupils Whose Teachers Consistently Asked More (E-group) or Less (D-group) than the Median Number at a Cognitive Level

(Number of Students in Parentheses)

Group	Items at Same Cognitive Level		Remaining Test Items	
	p-score	Z-value	p-score	Z-value
E-group memory	.62(55)		.38	
versus		2.74*		.81
D-group memory	.30(27)		.34	
E-group comprehension	.58(67)		.34	
versus		.15		-1.39
D-group comprehension	.57(71)		.39	
E-group application	.70(15)		.50	
versus		2.88*		2.44**
D-group application	.31(29)		.33	

*significant at the .01 level
**significant at the .05 level

For the comprehension level, there was no significant difference between p-scores of the groups for either comprehension items or the remaining test items.

For the application level, the p-score for the E-group on application items was significantly higher (at the .01 level) than the p-score of the D-group. The E-group's p-score for the remaining test items was also significantly higher (at the .05 level) than the D-group's p-score.

Conclusions

The present study has found a significant positive relationship between teacher emphasis of the memory and application levels and pupil achievement at those levels. The findings for the memory level indicate that differences in pupil achievement are probably not due to differences in pupil ability. However, the findings for the application level imply that the differences in pupil achievement at that level may be due to pupil ability rather than teacher behavior.

An alternative explanation for the findings at the application level is based on Bloom's conceptualization of his work. The taxonomy is a hierarchal ordering in which each level subsumes the lower ones. Thus the teacher who emphasizes the application level could be developing his pupils' ability to achieve at the memory and comprehension levels as well as the application level.

The data lend some support to this explanation. The p-score for the E-group on the memory and comprehension items is significantly higher at the .01 level ($Z = 2.67$) than the corresponding p-score for the D-group. However there is no significant difference between the p-scores of the two groups for the higher level items ($Z = .85$).

The present findings like those of any causal-comparative study do not permit one to infer a cause and effect relationship between teacher behavior and pupil achievement. This possibility is only one of several explanations (2). However many teacher-trainers have been urging their teachers to raise the cognitive level of their classroom questions (3). The present study gives them some empirical evidence to support their position.

References

1. Bloom, Benjamin S., (Ed.). Taxonomy of Educational Objectives: Handbook 1: Cognitive Domain, David McKay Co., New York, 1956.

2. Borg, Walter R. and Gall, Meredith D. Educational Research (2nd ed.), David McKay Co., New York, 1973.

3. Dunkin, Michael J. and Biddle, Bruce J. The Study of Teaching, Holt, Rinehart, and Winston, Inc., New York, 1974.

4. Farley, George T. Increasing the cognitive level of classroom questions: An application of Bloom's Taxonomy of Educational Objectives. Ed.D. dissertation, University of Massachusetts, 1968.

5. Freund, John E. Modern Elementary Statistics (3rd ed.), Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1967.

6. Friedman, Morton. A system to analyze geometry teachers' questions. Mathematics Teacher, December 1974, 67, 709-713.

7. Rogers, Virginia. Varying the cognitive levels of classroom questions in elementary social studies: An analysis of the use of questions by student teachers. Ph.D. dissertation, University of Texas, 1969.

8. Ragosta, Marjorie et. al. Sign versus category: Two instruments for observing level of thinking. Paper presented at the annual meeting of the American Educational Research Association, New York, 1971. In Dunkin and Biddle, The Study of Teaching.

THE INTERPRETATION OF THE CONNECTIVE OR IN DISJUNCTIVE ARGUMENTS

William A. Juraschek
The University of Texas at Austin

Purpose

The development of logical reasoning ability has been a continuing concern of educators. While Piaget (Inhelder & Piaget, 1958) has explored many aspects of this ability, several other studies (Jansson, 1973 (Review); O'Brien, 1973) have been concerned specifically with the understanding of propositional or "mathematical" logic in verbal form. Some studies (Ennis & Paulus, 1965; Gardiner, 1965; Roberge, 1970) have suggested that deductive ability improves with age, but the evidence is not conclusive. Roberge (1972) pointed out that the amount of improvement, if any, varies with the type of deductive principle. For example, ability to use correctly the law of detachment improves with age, but awareness of the fallacy of denying the antecedent shows no such improvement (Roberge, 1972). Likewise O'Brien (1973) found that college students did not perform equally well on all argument forms. As noted by Ennis (1975), one difficulty that permeates the literature on studies of deductive reasoning ability is the use of different instruments in different studies. Not all instruments use the same format and not all comprise the same argument forms. The present study focuses on one aspect of this problem: the interpretation of the connective or in disjunctive argument forms.

Neimark and Slotnick (1970) found with children in grades 3-9 that the connective and was understood well by all but third graders, but proficiency with the connective or increased with age. They also found that the wording -- whether stated "A or B" or "A or B or both" -- of the conditions had a significant effect on proficiency with this concept. Both Gardiner (1965) and Miller (1968) found improvement with age (grades 4-12) in proficiency with the argument forms:

I: P or Q. Not P. Therefore, Q.
II: P or Q. P. Therefore, no conclusion about Q possible.

Eisenberg and McGinty (1974), on the other hand, reported that on type I and II arguments, prospective elementary teachers did no better than second and third graders. In their study it was suggested that incorrect conclusions on type II arguments could have resulted from an assumption of exclusive disjunction for the connective or, rather than the required inclusive interpretation. In their instrument, (see Table 1) the type I and II items were stated using "Either...or" which, according to accepted usage, most often connotes an exclusive disjunction (Webster's 1964; Follett, 1966). Assumption of exclusive disjunction in type II arguments allows the valid conclusion "Therefore, not Q." Type I arguments are not affected by this ambiguity. Thus, it seems reasonable to suspect that errors with type II arguments may be due more to linguistics than logic.

The present study sought to answer three questions: 1) In type II arguments do students assume exclusive disjunction more often than inclusive disjunction? 2) In type II arguments do students assume exclusive disjunction more often when the phrase "Either...or" is used than when "or" alone is used? 3) In type II arguments do students assume inclusive disjunction more often when informed that this is the expected interpretation of the connective or?

Procedure

Three forms of a sentential logic test were randomly administered to 266 students enrolled in a mathematics course for prospective elementary teachers prior to any instruction in logic. Form A was identical to the instrument used by Eisenberg and McGinty (1974). The 30 items were divided into five groups of six items each. Twelve items tested understanding of the argument forms:

I: P or Q. Not P. Therefore Q.
II: P or Q. P. Therefore no conclusion about Q.

Eighteen items tested understanding of the argument forms:

III: If P, then Q. P. Therefore, Q.
IV: If P, then Q. Not Q. Therefore, not P.
V: If P, then Q. Q. Therefore, no conclusion about P.

All items were similar to the six type II items shown in Table 1. Each item could be answered Yes (definitely yes), No (definitely no), or Maybe (no definite conclusion).

Table 1

Portion of Sentential Logic Test*

Type II Items

---

2. Either Bill has a red ball or he has a blue bike.
   Bill has a red ball.
   Does Bill have a blue bike?

10. Janet either will not go to the game or she will not go to the race.
    Janet did not go to the race.
    Did Janet go to the game?

15. Mary either has a brother or she does not have a cousin.
    Mary has a brother.
    Does Mary have a cousin?

(continued on next page)

Table 1 (cont.)

19. Either Dick is not in the race or he is not in the game.
    Dick is not in the game.
    Is Dick in the race?

20. Either John is in room 4 or he is not in third grade.
    John is not in third grade.
    Is John in room 4?

27. Jack either has a brother or he has a sister.
    Jack has a brother.
    Does Jack have a sister?

---

*From Eisenberg and McGinty (1974)

Form B was the same as Form A with the exception that the word "either" was omitted from the type I and II items. Form C was the same as Form B, but the cover sheet contained the following note on the use of inclusive or:

> NOTE: The word or as used here is called inclusive or. This means that the sentence "John is tall or John is rich," is true when John is tall, when John is rich, and when he is both tall and rich. The sentence is false when John is neither tall nor rich. If the sentence "John is tall or John is rich" is true and the sentence "John is rich" is also true, one cannot conclude that John is not rich.

Results

As one can see in Table 2, for items of type III, IV, and V there was little variation in percent correct across test forms. The three mean

Table 2

Mean Percent Correct*

	Test Form		
Item Type	A (N=86)	B (N=90)	C (N=90)
I	70	60	51
II	12	22	42
III	96	96	95
IV	34	33	34
V	55	64	60

*Inclusive interpretation required.

percentages correct for the three item types are close to those (91%, 33%, 62% (by averaging)), reported by Eisenberg and McGinty (1974). The same is true of the percent correct for type I and type II items on Form A; however, as further analysis reveals, the extremely low percent correct on type II items was not necessarily due to faulty logic.

The data in Table 3 suggest affirmative answers to the first two questions posed in this study. First, students more often assigned an

Table 3

Percents Assuming Inclusive and Exclusive Or

Type II Items

Item #	Test Form					
	A		B		C	
	Inc.	Exc.	Inc.	Exc.	Inc.	Exc.
2	14	85	22	74	46	18
10	19	67	37	46	44	19
15	6	77	21	70	43	47
19	8	57	21	46	41	13
20	13	48	9	50	32	24
27	9	91	24	73	47	22

exclusive rather than an inclusive meaning to the connective or in type II arguments. The percent choosing a response consistent with an exclusive interpretation was significantly greater than that consistent with an inclusive interpretation for all six items in Form A and for all but item 10 of Form B ($\chi^2 > 7$, $p < .01$). Also, assuming exclusive disjunction on Forms A and B, these students gave a correct response an average of 71% and 65% of the time.

Second, comparing the data for Forms A and B, it appears that these students were more likely to choose the exclusive interpretation when the wording "Either...or" was used. Although only items 10 and 27 produced significant differences ($p < .05$) between the percents choosing the exclusive and inclusive interpretations, there is a definite trend, marred only by item 20. These results strongly suggest that the apparent lack of understanding of disjunctive arguments was at least partly due to linguistic factors rather than faulty logic.

An answer to the third question posed is not as immediate as those to the first two questions. Comparing Forms B and C, it appears that these students were more likely to assume the inclusive interpretation when informed of this meaning prior to testing. One can question this interpretation, however, on the grounds that the wording of the NOTE on inclusive

or that accompanied Form C made the "maybe" response more likely on both type I and II items. (Recall that the "maybe" response is correct for type II, but not for type I items.) Indeed, evidence supports this, since on type I and II items 38% chose "maybe" on Form C, while 22% did so on Form B, and these percentages are roughly the same for type I and type II items alone. It thus appears that a definite answer to the third question must await further research.

While this study has gathered further evidence that many college students lack understanding of several logical argument forms, it also has shown the subtle but important interaction between logic and language. It appears that unless instructed to do otherwise, a majority of students will assign an exclusive rather than an inclusive meaning to the ambiguous connective or. One should be cautious in judging the logical ability of students when they may be assigning a meaning to words different from that common to mathematics.

References

Eisenberg, T. A. and McGinty, R. L. On comparing error patterns and the effect of maturation in a unit on sentential logic. Journal for Research in Mathematics Education, 1974, 4(5), 225-237.

Ennis, R. H. Children's ability to handle Piaget's propositional logic: a conceptual critique. Review of Educational Research, 1975, 1(45), 1-40.

Ennis, R. H. and Paulus, D. H. Critical thinking readiness in grades 1-12 (phase 1: deductive reasoning in adolescence). U.S.O.E. Cooperative Research Project # 1680, Cornell University, 1965.

Follett, W. Modern American Usage. New York: Hill and Wang, 1966.

Gardiner, W. L. An investigation of understanding of the meaning of the logical operators in propositional reasoning. Unpublished doctoral dissertation, Cornell University. University Microfilms No. 664109, 1965.

Inhelder, B. and Piaget, J. The Growth of Logical Thinking from Childhood to Adolescence. New York: Basic Books, 1958.

Jansson, L. C. The development of deductive reasoning: a review of the literature. Paper presented at annual A.E.R.A. meeting, Chicago, 1973.

Miller, W. A. The acceptance and recognition of six logical inference patterns by secondary students. Unpublished doctoral dissertation, University of Wisconsin. University Microfilms, No. 6813651, 1968.

Neimark, E. D. and Slotnick, N. S. Development of understanding of logical connectives. Journal of Educational Psychology, 1970, (61), 451-460.

O'Brien, T. C. Logical thinking in college students. Educational Studies in Mathematics, 1973, 1(5), 71-80.

Roberge, J. J. A study of children's abilities to reason with basic principles of deductive reasoning. American Educational Research Journal, 1970, 4(7), 583-596.

Webster's Third New International Dictionary (unabridged). G&C Merriam Co., Springfield, Massachusetts, 1964.

INTERPRETATIONS OF LOGICAL CONNECTIVES BY PRE-SERVICE ELEMENTARY SCHOOL TEACHERS

Suzanne K. Damarin
The Ohio State University

Purpose

While the place of formal logic in the mathematics curriculum is the subject of some debate, it seems clear that the use of logical connectives and principles is inherent in the process of learning and doing mathematics. Logical processes are involved in proof and in deductive problem solving. Moreover, the definition and communication of mathematical concepts would seem to be dependent upon the use of assertions stated in the forms of standard logical propositions. An understanding of the abilities and predispositions which students (and teachers) exhibit in the handling of such statements is therefore of importance to the mathematics educator.

Analysis of research literature concerning logical abilities and *a priori* consideration of the domain of questions concerning these abilities suggest three types of questions one might ask in order to consider the extent to which students can use conditional logic:

1. What logical forms do students spontaneously exhibit in problem solving protocols (or other discourse)?

2. What valid (and invalid) inferences do students make from pairs or larger collections of statements?

3. What conditions of truth or falsity for simple statements do students accept as consistent with the truth of compound statements connecting them?

The first of these questions has been the subject of extensive work by Inhelder and Piaget (1959), whose claim that subjects are able to use propositional logic when and only when they reach adolescence, has been the subject of much debate in the recent literature (Ennis 1975, Weitz et. al. 1973, Bynum et. al. 1972 and others). The second question has been the focus of much educational and psychological research (Woodworth and Sells 1935, Hill 1961, O'Brien and collaborators 1968, 1960, 1971 and others), and several findings have been well replicated in this area.

The third question cited above concerns the behaviors of individuals in ascribing truth values to compound statements on the basis of truth or falsity of their constituent parts. Variations of this problem have been studied by several psychologists (Paris 1973, Suppes and Feldman 1971, Niemark 1970, and others). This problem is the focus of this paper.

The major purpose of the study was the determination of the extent to which pre-service elementary teachers use mathematical logic in their interpretations of compound statements in standard logical form. In particular the study examined how agreement with mathematical logic varies as a function of the connectives and "input" truth values of a statement.

The compound statements used in this investigation were intended to be as similar as possible to simple mathematical statements which pre-service elementary teachers must use in mathematics coursework and, later, in teaching. (Examples: "If n is even, then n + 1 is odd," and "2 divides p if and only if 4 divides p^2.") At the same time, for the purposes of the study, assignment of truth values to statements was to be independent of prior mathematical knowledge. In an effort to meet both of these requirements, items involving membership of symbolic elements in pictured sets were devised.

On the basis of related research, a small pilot project, classroom experience, and "folklore" several effects were expected. It was anticipated that subjects would exhibit near universal agreement with mathematical logic for conjunctive statements, and that a sizable proportion would use the "exclusive or" when interpreting the disjunction. The conditional and bi-conditional were expected to be more difficult (in the sense that fewer subjects would agree with mathematical logic), especially in cases where the first simple statement involved was false.

An additional purpose of the study was the determination of what changes in set membership subjects would deem necessary for changing the (assessed) truth value of the statements. In this regard the relative influences of the constituent parts of a compound statement, and related questions, were explored.

Procedure

The Truth Value Inventory (TVI), a thirty-two item instrument, was devised for this study. Each item of the TVI contains a single statement referring to a pair of sets pictured at the top of the page, and requires the subject to declare whether the statement is true or false. A second part of each item requires the subject to indicate a change (or changes) in set membership which would change the truth value of the statement. Two items were constructed for each cell in the 4 by 4 matrix shown in table 1.

The TVI was administered to all subjects (N = 70) in attendance on an unannounced date for a mathematics course. (Logic was not discussed in this course.)

Results

Responses were coded and submitted to analysis of variance. Significant effects ($p < .001$) were found for connectives, input truth values, and interactions between these. Differences in means were tested using Tukey's HSD.

Table 1

Matrix Used in Constructing TVI with Proportion of Responses in Agreement with Mathematical Logic Enetred in Each Cell

p	q	conjunction	disjunction	conditional	biconditional
T	T	.97	.92	.96	.89
T	F	.99	.39	.99	.99
F	T	.96	.49	.09	.91
F	F	.98	.97	.23	.24

As indicated in Table 1, the pre-service teachers tended to treat both conditional and biconditional statements as if they were conjunctive. Approximately half of the population treated disjunctive statements in this manner as well (only 4 used the "exclusive or").

The data related to moving objects were coded and combined with the true/false data to yield a nominal scale. Several hypotheses were accepted using the χ^2 statistic.

Further study of this problem, currently underway, includes use of an instrument differing from the TVI is that it allows for a "can't tell" response, and analysis of mathematics textbooks for pre-service elementary teachers in an effort to determine whether and where nonuse of mathematical logic would interfere with correct, or at least serviceable, interpretation of statements.

References

Bynum, Terrell Ward; Thomas, James A.; Weitz, Lawrence J. Truth-functional logic in formal operational thinking: Inhelder and Piaget's evidence. Developmental Psychology, 1972, 7, 129-132.

Ennis, Robert H. Children's ability to handle Piaget's propositional logic: a conceptual critique. Review of Educational Research, 1975, 45, 1-41.

Hill, Shirley A. A study of logical abilities in children. Unpublished doctoral dissertation, Stanford University, 1961.

Inhelder, B. and Piaget, J. The Growth of Logical Thinking from Childhood to Adolescence. Basic Books: 1958.

Niemark, Edith D. Development of comprehension of logical connectives: understanding of "or." Psychonomic Science, 1970, 21, 217-219.

O'Brien, Thomas C. Logical thinking in adolescents. Educational Studies in Mathematics, 1972, 4, 401-428.

O'Brien, Thomas C. and Shapiro, Bernard J. The development of logical thinking in children. American Educational Research Journal, 1968, 5, 531-542.

O'Brien, Thomas C.; Shapiro, Bernard J.; and Reali, Norma C. Logical thinking-language and context. Educational Studies in Mathematics, 1971, 4, 201-219.

Paris, Scott G. Comprehension of language connectives and propositional logical relationships. Journal of Experimental Psychology, 1973, 16, 278-291.

Suppes, Patrick and Feldman, Shirley. Young children's comprehension of logical connectives. Journal of Experimental Psychology, 1971, 12, 304-317.

Weitz, Lawrence J.; Bynum, Terrell Ward; Thomas, James A.; and Steger, Joseph A. Piaget's system of 16 binary operations: an empirical investigattion. The Journal of Genetic Psychology, 1973, 123, 279-294.

DIFFERENCES BETWEEN TEACHERS' ATTITUDINAL SELF-RATINGS AND THEIR PRINCIPALS' TEACHER IDEAL ON A MATHEMATICS INVENTORY

Thomas R. Post
University of Minnesota

William H. Ward, Jr.
University of Minnesota

Victor L. Willson
University of Minnesota

Purpose

This study was conducted to examine differences between mathematics teachers' self-ratings and their principals' "idealized" mathematics instructors on an inventory concerned with both the cognitive and affective outcomes of mathematics instruction. Individual items on the instrument were aggregated using factor analytic techniques into seven scales related to the teaching and learning of mathematics. Scale scores were compared using analysis of variance procedures.

Research comparing principal and teacher perceptions of the various dimensions of the in-school experience is sparse. An ERIC search conducted for this study did not generate a single research paper which specifically contrasted principal and teacher attitudes and/or perceptions. In a recent related study, Reineke and Welch (1973) examined the degree of heterophily (differences between individuals or groups of individuals) existing between principals and teachers regarding the perceived adequacy of school conditions. Results indicate that of the five dimensions examined (teacher effectiveness, teaching load, curriculum, facilities, and support staff) teaching effectiveness was considered most adequate among both groups. Their data also suggest that principals view conditions in their own schools considerably more favorably than do teachers. One of two possible explanations were suggested: (1) persons tend to view more favorably those conditions over which they have direct control (locus of control considerations) or (2) principals and teachers have become distinct groups through their selection, specialization of function, and professional interests, and that effective communication between them is lacking.

The study described herein is related in that principals' and teachers' attitudes and perceptions were again compared, this time as they relate specifically to the discipline of mathematics and its teaching. The identification of perceptual discrepancies between teachers and principals would appear to be an important first step in improving the educational environment of our schools.

Procedure

Principals in this study were selected randomly as part of an NSF grant to study change in selected regions of the country (see Welch and Gullickson,

1973). Stratifying variables were grade level (8th and 11th), urban and rural location, and subject matter (mathematics and science). Principals in California, Michigan, and Indiana were selected for the mathematics assessment. The principals selected one teacher at random from the mathematics faculty of his school. Both subsequently completed a battery of instruments. Among these was the Mathematics Inventory for Teachers (MIT) comprised of 30 items. Items on the MIT were taken from or were similar to items developed for the NLSMA (NLSMA Report 1968) or the International Study of Achievement in Mathematics (1967). The MIT assembled by Bracht (1972) contains statements concerning educational ideas and practices specifically dealing with the teaching and learning of mathematics in the secondary school. The items assess attitudes both toward mathematics as a subject and toward its teaching. It purports to let the respondent "express their beliefs and opinions." Each item is followed by the numerals 1 through 4 which correspond to: Strongly Agree, Agree, Disagree, and Strongly Disagree, respectively. Respondents are asked to circle the numeral which most closely corresponds to their reaction to the item statement. Principals were asked to respond to the MIT as they believed an ideal mathematics teacher should, while the randomly selected teachers were asked to express their own beliefs and opinions. A total of 148 teacher-principal pairs responded to this instrument.

The MIT originally was constructed to consist of five attitudinal subscales: mathematics as a process; classroom management practices; interest in teaching mathematics; rating of teaching practices; and rating of teacher concern for students. Both logical examination and inspection of scale coefficient alpha reliabilities convinced the authors that subscales were inadequate as constructed. The MIT was subjected to principal components analysis to extract new scales. Teachers' responses were used for the analysis, since they represent actual attitudes.

Eight factors were extracted from the principal components solution and were rotated using a varimax solution. The eight factors were examined for logical organization. Only items loading higher than .3 on the rotated solution were included in the factors. Of the eight, the first seven were conceptually identifiable. The last factor was considered to be a random factor.

Principal components analysis was chosen as the factoring procedure, since the factor scores would be easily interpretable. A common factors solution with R^2 as the communality estimate was examined, however, to see if the structure had stability. Eight factors were identified and rotated. The resulting structure was quite similar. (One factor split into two, but the factors were identifiable.)

The factors produced by the principal components rotated solution are identified and discussed below:

Factor 1 --

Scale items loading heavily on Factor 1 relate to a flexible perspective of both the teaching and learning of mathematics. Persons scoring high on

this factor should in their teaching tend to use a variety of instructional modes, as well as actively promote multiple approaches to the solution of mathematical problems. For these reasons, we have labeled Factor 1: FLEXIBILITY.

Factor 2 --

This factor centers around an interest and concern for the non-cognitive outcomes of mathematics teaching. It would appear also to include a recognition of the importance of the process dimension of effective mathematics learning. A high score on Factor 2 implies conscious attention to the development of positive student attitudes and an inherent belief that "mathematics is not a spectator sport." Factor 2 has been labeled: MATHEMATICS AS PROCESS.

Factor 3 --

MIT components clustering on this factor all relate to teachers' concern for student. A high factor score implies a strong professional commitment to the improvement of student achievement in mathematics and a willingness to provide additional instructional assistance when required. Factor 3 is labeled: TEACHER CONCERN FOR STUDENT.

Factor 4 --

Factor 4 is composed of items which relate primarily to the teachers' vocational satisfaction. Concern for the development of a productive learning environment is also a component. It should be noted that one item also loads significantly on Factor 2. This implies that various factors resulting from this analysis are not completely independent. Factor 4 is labeled: VOCATIONAL SATISFACTION.

Factor 5 --

MIT items loading on this factor generally are concerned with the development of precisely determined mathematical skills and are indicative of a more traditional and, certainly, a more conservative view of mathematics learning and teaching. Persons scoring high on Factor 5 have, at the very least, expressed an awareness of the existence of higher order teaching and learning objectives in the mathematics classroom. Two items load significantly on other factors. Factor 5 is labeled: NON-RIGID PRACTICES.

Factor 6 --

Labeled ATTITUDE TOWARD TEACHING, Factor 6 contains items conceptually related to those found loading on Factor 4, Vocational Satisfaction. This factor is not well defined and is considered to be one of the "weaker" factors resulting from this analysis.

Factor 7 --

Similar to Factor 5, Non-Rigid Practices, the five MIT items loading on Factor 7 are related to a concern for rules and precisely determined outcomes.

Two of the three items, which also load significantly on other factors, load on Factor 5. As was the case with Factor 5, persons scoring high on this factor have indicated disagreement with the positions stated. If translated into action, such disagreement would imply behaviors more consistent with the contemporary view as to appropriate objectives for mathematics learnings, that is, a recognition of the importance of higher order objectives. Factor 7 is labeled: HIGHER-ORDER CONCERNS.

Factor 8 --

Items loading on Factor 8 do not, in the opinion of the investigators, fall within a discernible conceptual framework. No attempt has been made, therefore, to either label or interpret this factor.

The actual MIT items comprising the various factors are available from the Minnesota Research and Evaluation Project.

Analysis

Scales were constructed from the seven factors using the procedure discussed by Harman (1967, p. 349). Scale reliabilities were estimated by weighting individual item scores by their corresponding varimax loadings. This procedure generates a conservative estimate of the true reliability coefficients. Subsequently, coefficient alpha was computed for the weighted responses, as noted by Armor (1974). These reliabilities and scale intercorrelations are given in Table 1.

Table 1

Reliabilities and Intercorrelations for Seven Scales Derived from the Mathematics Inventory for Teachers

	Flexibility	Math as Process	Teacher Concern	Voc. Sat.	Non-Rigid	Attde. Teach.	Higher Order
Flexibility	.57						
Mathematics as Process	.19	.51					
Teacher Concern	-.32	-.12	.49				
Vocational Satisfaction	.30	.33	-.22	.64			
Non-Rigid Practices	-.20	-.15	.29	-.16	.65		
Attitude Toward Teaching	-.20	-.01	.15	-.04	.09	.51	
Higher Order Concerns	-.05	-.29	.11	-.15	.47	.06	.57

Scale reliabilities are underlined on the diagonal.

Factor scale scores were computed for the principals' idealizations using the loadings observed for teachers' self-ratings. Under a null hypothesis of no difference between the principals' idealization and teachers' actual self-ratings, the scale scores computed this way should be expected to differ only randomly between teachers and principals.

Difference scores between each principal-teacher pair were analyzed with a seven dependent variable multivariate analysis of variance. Grade level (8 and 11) and region (California and Indiana-Michigan) were included as independent crossing variables. The primary test of interest is the test of the grand mean vector of differences, to see if it is non-zero in a multivariate sense. If so, univariate F-tests on the individual means will help to locate the differences.

Results

The multivariate analysis of variance was performed utilizing a program due to Finn (1970). The grand mean vector of differences was significantly different from zero in a multivariate sense at $p = .10$. Grade level, location, and the grade by location interaction were not significant at $p = .10$. Multivariate F-statistics for the grand mean and effects are given in Table 2, along with the univariate F-statistics for the grand mean.

Of the seven dependent variables, F-statistics for two were significantly different from zero, indicating principals and teachers produced different scores on Factor 3--Concern for Students, and Factor 7--Higher-Order Concerns.

The greatest disparity between principal and teacher scores was noted in Factor 3, Teacher Concern For Students. The significantly higher principal scores (recall that principals were asked to respond as they felt the ideal mathematics teacher would) implies that teachers, in general, are not as responsive to student needs, especially in the remediative sense, as their administrators would have them be. One can only speculate as to the underlying variables inherent in the observed discrepancy. Regardless of the reasons involved, however, it would seem that principals would have their teachers give more attention to those students who are having difficulty learning the subject. The data do not suggest that teachers are unaware of the importance of this function, nor do they suggest that teachers are wholly negligent, insofar as providing additional assistance to those in need is concerned. They do, however, suggest that, from an administrator's point of view, additional teacher efforts should be expended if all students are to maximize their individual potential.

One might also infer from the observed differences on Factor 3 scores that, from the principal's viewpoint, a disproportionate share of the teachers' attention is focused on those students at or above the acceptable level of mathematics achievement. If true, then pedagogical priorities need to be reassessed and/or redefined within the local setting. Such reassessment would inevitably lead to increased communication between principals and teachers. This should be beneficial to all concerned.

Table 2

Summary Table For Multivariate and Univariate
F-Statistics For Analysis of Difference Scores
Between Principals and Teachers on Seven Scale Scores

Multivariate Tests

Source of Variation	df source	df error	F-Statistic	Probability of Occurrence
Grand Mean	7	138	5.98	<.01
Grade	7	138	.65	<.71
State	7	138	.31	<.95
Grade x State	7	138	1.55	<.16

Univariate Tests for Grand Mean

df Grand Mean = 1 df error = 144

Variable (Factor)	MS Grand Mean	MS error	F-Statistic	Probability of Occurrence
1. Flexibility	.872	.523	1.67	<.20
2. Mathematics as Process	.205	.513	.40	<.53
3. Teacher Concern	12.54	.543	23.09	<.01
4. Vocational Satisfaction	.059	.627	.09	<.76
5. Non-Rigid Practices	.0003	.746	.0004	<.99
6. Attitude Toward Teaching	1.65	.728	2.27	<.14
7. Higher Order Concerns	2.37	.713	3.33	<.07

As can be seen from Table 2, the Principal-Teacher weighted scores on Factor 7 were also significantly different from zero. Three of the five MIT items within Factor 7 also load significantly on other factors. A low score on Factor 7 implies a narrow conception of the role and function of mathematics instruction, as well as a limited perspective as to the nature of the appropriate objectives. Principals scored lower on this scale, suggesting that teachers are more cognizant of the need for higher order outcomes in mathematics. Teachers expressed a greater degree of disagreement with MIT items loading significantly on Factor 7, which incidentally were negatively stated (i.e., strong disagreement implies a desirable viewpoint). Such disagreement is more in keeping with a "progressive" view and implies a recognition of the existence of a level of mathematics learning and achievement which transcends mere computational proficiency and direct applicability to everyday experiences. This outcome is not surprising since teachers have received more specialized training in both the subject and its pedagogy and should be more knowledgeable regarding such matters.

The more "conservative" position espoused by principals is also understandable. As the primary liaison between school and community and between school and district administrative offices, the principal is generally the first to be "held accountable" for student achievement within the school mathematics program and on standardized achievement tests. Among other things, standardized instruments often compare schools within districts, as well as individual mean school achievement to national norms. At best, such instruments rarely assess higher order objectives accurately and, at worst, assess no more than computational facility and other related student-conditioned responses. To the non-mathematics or non-science-oriented person who has, in all probability, received a relatively limited exposure to higher order mathematics learning objectives, it is conceivable that the negatively stated MIT items accurately depict the essence of the types of learnings measured by such tests and those stressed by many school mathematics programs. This, in the opinion of the writers, is unfortunately a relatively accurate assessment. This discussion is not meant to imply that such lower level objectives are not important. It is, however, felt that they must be kept within an appropriate perspective--that is, they are felt to be necessary but not sufficient.

Further, and perhaps a bit more disturbing (although understandable), such persons, because of their experiences with the subject, would tend to actually believe that Factor 7 items as stated depict both an acceptable approach to mathematics instruction and an appropriate description of its major objectives.

Conclusions

As indicated, weighted difference scores (Principal minus Teachers) were significantly different from zero in two of the seven identifiable factors. It is perhaps significant that such differences were not exhibited in more instances, given the fact that principals were asked to respond as their "ideal" mathematics teacher would, and teachers were asked to express their own beliefs and opinions. Lack of significant differences on five of seven

factors is probably due to one of two reasons: Either the power of the test is insufficient to discriminate between the two groups on the five variables, or there is general agreement among administrators and the mathematics teachers employed in their respective schools. If one assumes that teachers do, in fact, conduct mathematics classes in a manner consistent with their philosophical outlook, it follows that principals are generally satisfied with mathematics programs as they currently exist. The following comments are based on that assumption. Specifically, principals:

1. Agree that teachers exhibit an appropriate degree of classroom flexibility with respect to the pedagogical methods employed. (This statement results from no significant differences on Factor 1.)

2. Are in accord with the degree of attention paid to the process outcomes (attitudes, interests, and the development of general problem solving techniques) of mathematics instruction. (This statement results from no significant differences on Factor 2.)

3. Believe that teachers are satisfied with their vocational choice. (This statement results from no significant differences on Factor 4.)

4. Are in agreement as to the amount of time and effort expended to promote student mastery of highly specific mathematical rules, skills, and procedures. (This statement results from no significant differences on Factor 5.)

It is unlikely, in light of the degree of satisfaction indicated above, that suggestions for substantive modifications of existing mathematics curricula from sources outside the school would be well received. If true, this would ultimately prove to be a major impediment to curricular innovation and may, in fact, account for the agonizingly slow pace with which new and significantly different curricular materials, objectives, and instructional procedures are accpeted and implemented within the school setting. Studies are needed to determine the degree of difference between the perspectives of school personnel (changees) and university professors (change initiators) as these perspectives relate to the appropriate nature of the teaching and learning of mathematics. Should large scale differences exist, further light would be shed on the nature and difficulties inherent in promotion of mathematics curricular innovation. Such studies are currently projected by the Minnesota Research and Evaluation Project (MREP).

References

Armor, D. J. Theta reliability and factor scaling. Sociological Methodology 1973-74 (ed. by H. L. Costner), San Francisco: Jossey-Bass, 1974.

Bracht, G. H. Description of the Mathematics Attitude Inventory for Teachers and copy of the Mathematics Inventory for Teachers (MIT) instrument, Project Report No. 1, NSF Research Project, University of Minnesota, 1972.

Finn, J. D. "Program Multivariance," mimeo, 1968.

Harman, H. H. Modern Factor Analysis, 2nd ed., Chicago: University of Chicago Press, 1967.

International Study of Achievement in Mathematics, Vols. I and II (ed. by Torsten Husen), New York: John Wiley and Sons, 1967.

Reineke, R., and Welch, W. Adequacy of teaching conditions as perceived by administrators and teachers, Research Paper No. 7, Minnesota Research and Evaluation Project, University of Minnesota, 1973.

Wilson, J.; Begle, E.; and Leonard, C. National Longitudinal Study of Mathematics Achievement (NLSMA) Report No. 1, Part A, B, "X-Population Test Batteries," SMSG, 1968.

RESEARCH REPORTING SECTION #2

Presider: JOE CROSSWHITE, The National Science Foundation
Washington, D.C.

Reports: INTUITIVE EQUATION SOLVING SKILLS AND THE EFFECTS ON THEM OF FORMAL TECHNIQUES OF EQUATION SOLVING

BETSEY S. WHITMAN, Florida A and M University
Tallahassee, Florida

THE EFFECT OF INSTRUCTION IN HEURISTICS ON THE ABILITY TO SOLVE OPEN ENDED MATHEMATICAL PROBLEMS

LIONEL PEREIRA-MENDOZA, Memorial University
St. John's Newfoundland, Canada

CONCEPTS FOR MATHEMATICS TERMS COMPARED BETWEEN INDIAN STUDENTS AND THEIR NON-INDIAN PEERS

DOUGLAS G. GARBE, Brigham Young University
Provo, Utah

THE EFFECT OF EXPLORATORY AND REVIEW HOMEWORK EXERCISES UPON ACHIEVEMENT, RETENTION, AND ATTITUDE IN A FIRST-YEAR ALGEBRA COURSE

CHARLES D. FRIESEN, The University of Nebraska-Lincoln
Lincoln, Nebraska

INTUITIVE EQUATION SOLVING SKILLS AND THE EFFECTS ON THEM OF FORMAL TECHNIQUES OF EQUATION SOLVING

Betsey S. Whitman
Florida A and M University

Purpose

This study was designed to assess the merit of intuitive equation solving techniques and to determine the effects of subsequent formal techniques of equation solving. The investigations' thrust was toward determining what effect the learning of formal techniques had on the intuitive ability students had gained from instruction.

Intuitive techniques refer to techniques for solving equations that depend on asking appropriate questions when confronted with an equation, answering the questions in sequence, and recording the roots for the equation. To solve the equation $69 - \frac{96}{7 - \Delta} = 37$, a student might interact with himself mentally as follows:

Question	Response
Sixty nine minus what number gives 37?	32
Ninety six divided by what number is 32?	3
Seven minus what number is 3?	4
Solution: 4	

A student taught to solve equations by formal techniques, in applying them to the same equation, would write a sequence of steps like the following:

Solution:

1. $69 - \frac{96}{7 - \Delta} = 37$
2. Multiply by $(7 - \Delta)$, the L.C.D. $\quad (7 - \Delta)69 - 96 = (7 - \Delta)37$
3. Remove parentheses $\quad 483 - 69\Delta - 96 = 259 - 37\Delta$
4. Combine terms $\quad 387 - 69\Delta = 259 - 37\Delta$
5. Subtract 259 $\quad 129 - 69\Delta = -37\Delta$
6. Add 69Δ $\quad 128 = 32\Delta$
7. Divide by 32 $\quad 4 = \Delta$

Check:
$$69 - \frac{96}{7 - 4} = 69 - \frac{96}{3}$$
$$= 69 - 32$$
$$= 37$$

The solution is 4.

The several questions posed were these:

(a) Can students who have had no instruction in equation solving come up with correct solutions when faced with a set of equations to solve and therefore need no instruction?

(b) Is it possible to improve students' intuitive ability through instruction?

(c) To what extent do students who gain intuitive skills through instruction retain these skills after resting from equation solving?

(d) Does instruction in formal techniques of equation solving frustrate students' intuitive skill?

(e) To what extent can initial intuitive skill, once gained, be enhanced through additional instruction?

Procedure

To investigate these questions, an experiment was conducted which was designed to yield data concerning the subjects' initial ability to solve equations, their skill changes made through instruction, and their retention of skills acquired.

Subjects were 156 seventh grade students from four classes (two level 4, two level 3) at Cobb Middle School and from two classes at the Developmental Research School (DRS), both in Tallahassee, Florida.

The six intact classes were given a 48-item pretest. Students in each class were ranked on the basis of the average of their pretest and IQ scores' ranks. Based on these ranks, each class was separated into two matched groups. Thus, there were twelve groups, four in the Cobb level 4 classification, four in the Cobb level 3 classification, and four in the DRS classification. In each of these classifications, one group was designated treatment 1, one treatment 2, one treatment 3, and one treatment 4.

The treatment instruction began on the day after the pretest. Groups in treatments 1, 2, and 3 were taught intuitive techniques of equation solving for eight days. The groups in treatment 4 did no equation solving during this time. Then, all students took posttest I.

On the next day, students in treatments 3 and 4 began the studying of formal equation solving techniques, those in treatment 2 began a unit involving no equation solving, and those in treatment 1 continued their study of intuitive techniques. This phase of the study continued for seven class days. Then, students took posttest II.

Results

The data analyses involved three analyses of variance which were performed on the difference scores from pretest to posttest I, from posttest I to posttest II, and from pretest to posttest II. The Wilcoxon matched-pairs signed-ranks test was used on ratios of raw scores of posttest II to those of posttest I.

The results of the third analysis of variance and a subsequent Newman-Keuls test supported six of the original hypotheses. Those that were supported were:

1. Different kinds of instruction in equation solving (intuitive and formal) given in different combinations and for different lengths of time produce different gains in equation solving ability.

2. Students who learn to solve equations only intuitively perform better than those who learn both ways in close proximity.

3. After initial gains in equation solving ability with intuitive instruction, more intuitive instruction improves students' ability in equation solving.

4. Students who have instruction in both techniques of equation solving perform better than those who only have instruction for one-half as long and only in formal techniques.

5. Students gain more equation solving ability when they are taught only intuitive techniques and then rest from equation solving than they do when taught only formal techniques, in nearly equal instruction time periods.

6. With about twice as much instruction, students who learn only intuitive equation solving perform better than those who learn only formal techniques for one-half as long.

The Wilcoxon test supported the hypothesis that the ratios of posttest II to posttest I in treatment 2 are higher than those in treatment 3. The support of this hypothesis was strong evidence for the original conjecture that formal techniques tend to thwart students' intuitive ability to solve equations. Indeed, its support indicated that no equation solving at all is better than learning formal techniques after intuitive ones.

Recommendations

The results of this study led to some answers to the original questions. For the question of whether students have innate intuitive skill for solving equations and therefore need no instruction, the large number of students (35.5 percent) who solved fewer than five out of 48 equations on the pretest indicated that most students need instruction to learn to solve equations.

For the question of whether students' intuitive ability to solve equations could be improved through instruction, the study found a definite positive response. In every case, the group means of students who learned intuitive techniques essentially doubled after eight days of instruction.

The third question was about the amount of retention students would exhibit after resting from equation solving. The change in this group was a gain rather than a loss. The answer to the fourth question, whether instruction in formal techniques frustrated or thwarted students' intuitive skill, appeared to be a definite yes. The analyses on both differences and ratios of change from posttest I to posttest II showed that the groups who learned formal techniques were hindered in their progress.

An answer to the last question, about the enhancement of intuitive skill through more instruction, was found by considering the increase of scores of students in treatment 1 from posttest I to posttest II. These students had an average score of 16.3 (out of 48 possible) on posttest I and an average score of 23.3 on posttest II.

THE EFFECT OF INSTRUCTION IN HEURISTICS ON THE ABILITY TO SOLVE OPEN ENDED MATHEMATICAL PROBLEMS

Lionel Pereira-Mendoza
Memorial University

Purpose

The study investigated whether it was possible to teach students to apply at least one of the heuristics of examination of cases or analogy to novel mathematical problems, and whether learning to apply heuristics can be better accomplished by having students focus solely on the heuristics, rather than also involve them with the simultaneous learning of mathematical content. Two factors were considered in the design of the study. The first factor concerned employing treatments designed to meet different objectives: one instructional treatment was designed to teach heuristics only (H), another combined the instruction in heuristics in the teaching of content (HC), and a third treatment utilized content only (C), without reference to heuristics. The latter treatment was used as an experimental control. The second factor concerned employing distinct instructional vehicles for the treatments, namely algebraic, geometric and mathematically neutral.

Many writers (for example, Rosenbloom, 1966; Richards, 1968; Bernard, 1972) have indicated that teaching students techniques for acquiring new knowledge is a major objective of the educational process. Polya (1957, 1962, 1965) is preeminent among mathematics educators in promoting the heuristic process as a means of teaching students how to approach and solve novel mathematical problems. Studies, such as those by Ashton (1962), Libeskind (1971), Lucas (1972) and Goldberg (1974) all based their instructional procedures on Polya's heuristic process. All four studies found evidence to support the hypothesis that instruction involving heuristics will improve problem solving performance. Furthermore, Lucas's results, while limited due to factors such as a small sample size (30 students), indicated the possibility of students transfering heuristics to test problems. Studies by Kersh (1964), Worthen (1965) and Wills III (1967) all provided evidence that students can search for patterns when faced with new problems.

In the present study two specific heuristics were taught, analogy and examination of cases. These heuristics play a significant role in Polya's conception of the heuristic process.

Procedure

Materials: Nine groups were utilized in the study corresponding to each of the treatment x vehicle combinations. Self-instructional booklets were developed for each group. Each of the booklets dealt with three topics, with three days being allotted to each topic and one additional day for review.

Instruments: Two tests were developed, one algebraic and the other geometric. The instruments both followed identical format, consisting of a novel mathematical problem (developed specifically for the study) followed by an open ended question:

What can you find out about...?

For example, the algebraic test consisted of presenting subjects with two operations modulo 7: 'U?-one multiplication' - a @ b = a x (b + 1), and 'Double addition' - a # b = 2 x (a + b) and asking subjects:

What can you find out about these operations?

Grading Scheme: The grading scheme for the tests consisted of assigning a subject's response to one of six categories. These were: 0 - No evidence of either examination of cases or analogy; 1 - Examination of cases only and no attempt to look for a pattern; 2 - Examination of cases and looking for a pattern; 3 - Analogy only; 4 - Both heuristics used distinctly; and 5 - Both heuristics used together.

Subjects: The subjects were 294 grade ten boys in an all male high school in Canada. Subjects were randomly assigned to one of the nine experimental groups. The day prior to subjects starting their respective treatments a note was read to all subjects explaining that they were about to take part in an experiment. On the day immediately following the ten day instructional period (one class period per day) the algebraic test was administered, and the geometric test administered the next day.

For a subject's test to be graded he had to be present for at least nine of the ten instructional periods. A team of three judges graded the tests individually and then a series of meetings were held to discuss tests on which there were disagreements. Tests on which all three judges could not agree were dropped from the study. The percentage of agreement on the algebraic test was 98% and on the geometric test 96%.

Analysis

A subset of the tests on which the judges agreed was used in the analysis. Subjects from whom data from only one test was available were discarded. Then additional subjects were eliminated (randomly, from appropriate groups) in order to equalize the numbers in each group and obtain an orthogonal design. Thus data from 189 subjects (21 per group) were used in the analysis. Parallel analysis of variance were performed on the two tests. In order to perform the primary analysis the data were recategorized 0 - 1; with 0 representing no heuristics (original 0) and 1 at least one heuristic (original 1,2,3,4 and 5). This was necessary since the original categorization scheme does not represent either an underlying continuum or a totally ordinal scale, and in that form the data were inappropriate for analysis by ANOVA.

Results

The analysis enabled the investigator to conclude that it was possible to teach students to apply at least one heuristic to novel mathematical problems. In particular, the H treatment was significantly better than the C treatment on both tests. The analysis indicated that the HC treatment was successful in teaching subjects to apply at least one heuristic to a novel geometric problem, but not for the algebraic problem.
The investigator speculated that the different results concerning the effectiveness of the HC treatment on the two tests may be attributable to the HC treatment better equipping subjects to apply examination of cases than to apply analogy, and this manifested itself on the geometric test where the problem was more amenable to the application of this heuristic.

Finally, an exploratory nonparametric analysis (χ^2) indicated that subjects who were in the top third on either reading or mathematical achievement (or both) tended to benefit most from the H treatment.

References

Ashton, Sister M. R. Heuristic methods in problem solving in ninth grade algebra. (Doctoral dissertation, Stanford University) Ann Arbor, Michigan: University Microfilms, 1962, No. 62-02320.

Bernard, H. W. Psychology of Learning and Teaching. (3rd ed.) New York: McGraw-Hill, 1972.

Goldberg, D. J. The effects of training in heuristic methods on the ability to write proofs in number theory. Paper presented at the annual meeting of the National Council of Teachers of Mathematics, Atlantic City, April, 1974.

Kersh, B. Y. Directed Discovery Vs. Programed Instruction: A Test of a Theoretical Position Involving Educational Technology. Oregon State System of Higher Education, Monmouth, Oregon, 1964.

Libeskind, S. A development of a unit on number theory for use in high school, based on a heuristic approach. (Doctoral dissertation, University of Wisconsin) Ann Arbor, Michigan: University Microfilms, 1971, No. 71-25201.

Lucas, J. F. An exploratory study on the diagnostic teaching of heuristic problem solving strategies in calculus. (Doctoral dissertation, University of Wisconsin) Ann Arbor, Michigan: University Microfilms, 1972, No. 72-15368.

Polya, G. How To Solve It. New York: Doubleday Anchor Books, 1957.

Polya, G. Mathematical Discovery. Volume I. New York: Wiley, 1962.

Polya, G. Mathematical Discovery. Volume II. New York: Wiley, 1965.

Richards, D. K. Curriculum development at the local level. The Bulletin of the National Association of Secondary School Principals, 1968, 52, 38-47.

Rosenbloom, P. Problem making and problem solving. In The Conference Board of the Mathematical Sciences (Eds.), The Role of Axiomatics and Problem Solving in Mathematics. Boston: Ginn, 1966.

Wills III, H. Transfer of problem solving ability gained through learning by discovery. (Doctoral dissertation, University of Illinois) Ann Arbor, Michigan: University Microfilms, 1967, No. 67-11937.

Worthen, B. R. Discovery vs. expository sequencing in six weeks of classroom instruction. In Della-Piana, G. M. et. al., Sequence Characteristics of Text Materials and Transfer Learning. Bureau of Educational Research, University of Utah, Part I, 1965 (a); Part II, 1965 (b).

CONCEPTS FOR MATHEMATICS TERMS COMPARED BETWEEN INDIAN STUDENTS AND THEIR NON-INDIAN PEERS

Douglas G. Garbe
Brigham Young University

Purpose

An examination of the literature shows a rather disheartening picture of the academic standing and the success experienced by the American Indian student in school. One should not falsely assume, however, that Indian students do not have the potential to succeed academically. Speculation as to reasons for the Indian student's poor achievement in school introduces many variables, some of which researchers have not yet studied.

Mathematics as an academic area ranks among the lowest in reported success for the Indian student. Little attempt has been made through research to reveal his sources of difficulty in this subject. The study to be reported was an initial attempt to identify at least one possible source of difficulty for the Indian student in his study of mathematics.

The study was designed to answer the following two questions:

(1) Do Indian and non-Indian students have different concepts for familiar basic mathematics terms as indicated by their selection of a meaning for a term when given a choice of meanings?

(2) Do Indians attending an all-Indian school and Indians attending a 30-60% Indian school have the same concepts for familiar basic mathematics terms as indicated by their selection of a meaning for a term when given a choice of meanings?

Procedure

Two hypotheses were formulated to answer the questions of the study. To obtain the necessary empirical data to test the hypothesis related to the first question an instrument had to be developed. The instrument contained mathematics terms taken from those reported as familiar to eighth grade students by Byrne and Kane (1971). Developmental work and preliminary testing provided the information necessary to offer under each term in the instrument two choices for a meaning in a multiple choice format. These two choices were designed to be of such a nature that one choice would be more frequently selected by Indian students than by non-Indian students. The hypothesis related to the second question was to evaluate the influence of the school setting as a variable.

The subjects of the study were the eighth grade students from three rural schools in the same geographic region. One school had an all-Indian population, the second school had approximately a 50 percent Indian student population, and the third school had an all-white population. Indian

students were those classifying themselves as such and indicating that an Indian language is spoken by at least one parent. The Indian students in the study were Navajo. The non-Indian students were Caucasion (white).

Results

In April of 1973 the instrument of the study was administered to 127 eighth grade Indian students and to 130 eighth grade non-Indian students. The data obtained were analyzed using the Kolmogorov-Smirnov two-sample test. The first hypothesis was tested by comparing the distribution of scores of the Indian students with those of the non-Indians. The statistic generated for this hypothesis was significant at the .001 level which strongly indicated that Indians and non-Indians do have different concepts for the basic mathematics terms of the instrument. The second hypothesis was tested by comparing distributions of scores of Indians from the all-Indian school with Indians from the 30-60 percent Indian school. The statistic generated for the second hypothesis failed to be significant at the .05 level which indicated that Indian students do not have different concepts for the mathematics terms on the instrument regardless of the school they attend.

The Kolmogorov-Smirnov test was also used to make comparisons of distributions of scores for six subscales of items of the instrument. Resulting from the examination of the subscales of items a tentative conclusion reached was that Indian students preferred a verbal statement expressing order rather than a symbol as a meaning for a number word. By examining individual items on the entire instrument an observation was that Indian students seemed to prefer generalities more so than non-Indians. Furthermore, Indians in greater proportion than non-Indians seemed to favor a more common everyday usage of terms rather than a mathematical usage.

Recommendations

One limitation of the study was that only Navajo students made up the Indian sample so care must be exercised in generalizing to all Indians. The investigator is currently analyzing results from a study conducted in May of 1975 where the Indian sample was eighth grade Ute Indians.

Some implications from Navajo study are as follows: (1) Teachers can be informed that Indians (Navajo) and non-Indians have different concepts of basic mathematics terms and thus try to make certain a term is conceptualized by all students in the same way before it is used in building definitions, providing instruction, or developing other concepts. (2) A list of terms specific to mathematics could be provided English teachers of Navajo children so mathematics and English teachers could better coordinate their efforts for the benefit of the Indian students. (3) If Indian students like generalities in preference to specifics in mathematics, modification could be made in teaching strategies so estimation techniques may be utilized initially with refinements made later to the more precise.

The study although exploratory in nature provides enough evidence to warrant further research. Such research can provide significant information to be used on the reevaluation of materials, objectives, and teaching strategies. The necessary modifications could then be made to help Indian students attain a level of excellence in mathematics in accordance with the investigator's prediction of their potential based on his own personal belief and his teaching experience.

References

Byrne, M. A. and Kane, R. B. Measuring vocabulary and symbol familiarity in the language of mathematics. Paper presented at the annual meeting of the American Educational Research Association, New York, February 1971.

Garbe, D. G. Indians and non-Indians of the Southwestern United States: comparison of concepts for selected mathematics terms. Unpublished doctoral dissertation, The University of Texas at Austin, 1973.

THE EFFECT OF EXPLORATORY AND REVIEW HOMEWORK EXERCISES UPON ACHIEVEMENT, RETENTION, AND ATTITUDE IN A FIRST-YEAR ALGEBRA COURSE

Charles D. Friesen
The University of Nebraska-Lincoln

Purpose

The purpose of this study was to investigate the relative effectiveness of two homework organizational patterns. Peterson (1971) has described four such patterns in detail. This study compared the vertical and the oblique homework organizational patterns.

A Vertical Homework Organizational Pattern is a pattern of assigning homework that includes only exercises that directly relate to the topic discussed the day the assignment is made. This homework assignment does not include consciously-planned review or exploration of material. An Oblique Homework Organizational Pattern is a pattern of assigning homework in which the homework a) explores material to be included in future class presentation, b) provides practice on the topic discussed the day the assignment is made, and c) reviews material presented in previous class sessions.

This study holds significance for the areas of student learning, classroom instruction, and curriculum development. If the oblique pattern improves student achievement and retention, these findings would be of significant interest to all persons concerned with improving student learning in the classroom. Practices of classroom teachers, methods taught in teacher pre-service and in-service classes and homework exercises appearing in mathematics textbooks should all reflect the results of this study.

Hovland (1951) has presented evidence that retention of meaningful material dissipates rapidly following learning. Ausubel (1968) has suggested that short and widely spaced review sessions that are introduced progressively further apart are best for meaningful learning and retention. Periodic or spaced reviews have been identified as playing an important role in retention by Gagne (1970). Among the many mathematics educators advocating the review of previously learned concepts are Butler and Wren (1965), Lankford(1959), and Dodes (1953). Laing (1970) and Klinger (1973) have shown homework review exercises to facilitate mathematical learning.

The opportunities for students to explore and discover are also highly valued by mathematics educators. Many of the exercises in the UICSM materials were designed to foreshadow topics encountered at a later time (Beberman and Vaughn, 1964). The Cambridge Conference on School Mathematics strongly emphasized that every opportunity be taken to let the students explore mathematical situations. Riedesel (1971) has noted that in most studies conducted at the elementary school level an increased amount of exploration time resulted in better understanding and retention of the topic. Peterson (1969) has reported that exploratory homework exercises improved student achievement and retention.

Procedures

This study provided evidence relating to whether assigning exploratory and review exercises on a topic would improve the achievement, retention, and attitude of first-year algebra students. Two treatment groups were compared. The vertical homework assignments of the control group (Group C) on a given day consisted of exercises relating only to the topic taught that day. The oblique homework assignment of the experimental group (Group E) differed in two respects. First, Group E received fewer exercises relating to the topic taught that day than did Group C. Secondly, Group E received a) exploratory exercises on each of the two days prior to the teaching of a topic, and b) review exercises on the first and third days after the teaching of a topic. A total of 143 first-year algebra students enrolled in three Lincoln, Nebraska, public junior high schools constituted the sample. Each of three teachers taught an experimental and a control class. The instructional material for the study consisted of three consecutive chapters of a first-year algebra test.

The pretest scores on The Revised Math Attitude Scale were used as covariates in analyzing the posttest scores. The Mid-Year Algebra Test was administered prior to the study and used as a covariate in the analysis of student achievement. The dependent measures of achievement consisted of three chapter tests and a retention test. The latter was administered four weeks after the administration of the third chapter test. The chapter tests and the retention test were designed by the researcher and evaluated for content validity by a panel of four mathematics educators. All of the tests were multiple-choice instruments and the Kuder-Richardson Formula 20 estimates of reliability were .82, .74, .81, and .80.

The treatment groups were compared on a) overall achievement, b) initial learning, as measured by the three chapter tests, c) retention, and d) attitude. Since the two treatments were used in each of three schools, a 2 x 3 design was used to analyze the results. The .05 level of significance was used.

Results

The results of a multivariate analysis indicated a significant difference ($p < .012$) in overall achievement between Groups E and C, favoring Group E. The interaction between treatments and schools was not significant.

Comparisons between Groups E and C on the chapter tests favored Group E in two of the three cases. The differences between the adjusted means of the two groups favored Group E on the first and third chapter tests (by 1.19 and .54, respectively). On only the first chapter test were the two groups significantly different ($p < .033$). The difference between the adjusted means of the second chapter test favored Group C by .07 and was not significant.

The greatest positive effect of the review and exploratory exercises was found in the results of the retention test. Group E students scored significantly higher ($p < .0034$) than did Group C students. The Group E adjusted mean on the twenty-seven problem retention test was 15.05, while the Group C mean was 12.94.

The experimental treatment had neither a positive nor a negative effect on student attitude toward mathematics. The mean attitude scores of the two groups were not significantly different.

A homework opinionnaire revealed no significant differences in the responses of the two groups. Group E students did not view the role played by homework assignments in the process of learning mathematics any differently than did Group C students. However, Group E students viewed the exploratory and review exercises as helpful and wanted their teachers to continue to use them.

References

Ausubel, David P. Educational Psychology: A Cognitive View, New York: Holt, Rinehart and Winston, Inc., 1968.

Beberman, Max and Vaughn, Herbert E. High School Mathematics, Boston: D. C. Heath and Co., 1964.

Butler, Charles H. and Wren, F. Lynwood. The Teaching of Secondary Mathematics, 5th edition, New York: McGraw-Hill Book Company, 1970.

Cambridge Conference on School Mathematics. Goals for School Mathematics, Boston: Houghton Mifflin Company, 1963.

Dodes, Irving Allen. Planned instruction. The Learning of Mathematics: Its Theory and Practice, pp. 303-334, Twenty-first Yearbook of the National Council of Teachers of Mathematics, Washington, D.C.: NCTM, 1953.

Gagne, Robert M. Some new views of learning and instruction. Phi Delta Kappan, May 1970, 51, 468-472.

Hovland, C. I. Human learning and retention, in S. S. Stevens (Ed.), Handbook of Experimental Psychology, New York: Wiley and Co., 1951, 613-689.

Klinger, William R. Effect of distribution of earlier concepts as preliminary homework exercises upon achievement in a remedial mathematics course at the college level. Unpublished Doctoral Dissertation, The Ohio State University, 1973.

Laing, Robert A. Relative effects of massed and distributed scheduling of topics on homework assignments of eighth grade mathematics students. Unpublished Doctoral Dissertation, The Ohio State University, 1970.

Lankford, Francis G. Implications of the psychology of learning for the teaching of mathematics. The Growth of Mathematical Ideas, K-12, pp 405-430, Twenty-fourth Yearbook of the National Council of Teachers of Mathematics, Washington, D.C.: NCTM, 1959.

Peterson, John C. Effect of exploratory homework exercises upon achievement in eighth grade mathematics. Unpublished Doctoral Dissertation, The Ohio State University, 1969.

Peterson, John C. Four organizational patterns for assigning mathematics homework. School Science and Mathematics, October, 1971, 71, 592-596.

Riedesel, C. Alan. Research suggestions: Use of time in teaching elementary school mathematics. The Arithmetic Teacher, March, 1971, 18, 177-179.

RESEARCH REPORTING SECTION #3

Presider: MERLYN J. BEHR, Florida State University
Tallahassee, Florida

Reports: A STUDY OF THE YOUNG CHILD'S ABILITY TO USE PROPERTIES OF THE MEASUREMENT FUNCTION

CHARLES E. LAMB, The University of Texas at Austin
Austin, Texas

A DESCRIPTION OF THE DIFFICULTIES SELECTED EIGHT YEAR OLD CHILDREN HAVE IN LEARNING THE DISTRIBUTIVE ALGORITHM FOR DIVISION

LINNEA WEILAND, Kean College of New Jersey
Union, New Jersey

ROBERT MADELL, Village Community school
New York, New York

THE EFFECTS OF PRECISE THINKING ALGORITHMIC EXERCISES ON THE FIFTH GRADE STUDENT'S ABILITY TO PERFORM THE DIVISION ALGORITHM

GILBERTO JOSE CUEVAS, University of Miami
Miami, Florida

IMPLEMENTATION OF AN INDIVIDUALIZED APPROACH WITH COLLEGE FRESHMAN MATHEMATICS SURVEY COURSES: A STUDY

THOMAS A. EGAN, West Chester State College
West Chester, Pennsylvania

JOHN J. KERRIGAN, West Chester State College
West Chester, Pennsylvania

MARK F. WIENER, West Chester State College
West Chester, Pennsylvania

C. RALPH VERNO, West Chester State College
West Chester, Pennsylvania

A STUDY OF THE YOUNG CHILD'S ABILITY TO USE PROPERTIES OF THE MEASUREMENT FUNCTION

Charles E. Lamb
The University of Texas at Austin

Purpose

The field of measurement may be considered from two different viewpoints: (1) the viewpoint of a scientist using it as a tool, and (2) the viewpoint of a mathematician considering the measurement system as a mathematical model. In particular, the scientist would be involved with observational processes in making a determination about the real world, whereas the mathematician would consider the measurement system as a consistent and well-organized, intact mathematical entity. Children need to be familiar with the measurement process at both levels of consideration. The observational process serves as a foundation for the practical application of measurement in everyday life. whereas the intact system of measurement provides a theoretical look at the way mathematics develops and serves as a basis for future mathematical learning. In short, the child needs an understanding of measurement to quantify his world, -- he must be able to give both verbal and mathematical descriptions of his environment (Osborne, 1974).

Technically, measurement can be considered as a function. The domain of the function consists of objects to be measured and the range is some mathematical structure (usually a subset of the real numbers). The domain of the function has structure provided by the comparison of objects in the domain. By employing empirical processes, relations are operationally defined. These processes are either physical or abstract comparisons used to "measure" an object relative to another. By a physical comparison, one could consider an activity such as laying sticks side by side to determine if one is longer than the other. An abstract comparison would be a child making a decision concerning two numbers (used to describe an object or a collection of objects; Carpenter, 1971). The present study investigated the child's use of the transitive and substitutive properties of relations using solely physical comparisons and in a context requiring both physical and abstract comparisons. That is, the present study looked at children's ability to operate with aspects of both the domain and range of the measurement function.

Procedure

Each child received a series of twenty-eight tasks, seven each in four categories. The four categories were: (1) matching without numerical cues (abstract comparisons); (2) matching with numerical cues; (3) length without numerical cues; and (4) length with numerical cues. These tasks were designed using the following seven paradigms:

(1) If A = B and B = C, then A = C.
(2) If A < B and B < C, then A < C. — Transitivity
(3) If A > B and B > C, then A > C.

(4) If A = B and B < C, then A < C.
(5) If A = B and B > C, then A > C. — Substitution
(6) If A < B and B = C, then A < C.
(7) If A > B and B = C, then A > C.

With each task the child was asked to give a verbal explanation for his response. Responses were categorized as 0, 1, or 2 corresponding to no response, response without justification, and response with justification.

Analysis

The scores were analyzed with the use of a categorical scaling technique (item-analysis; Kundert and Bargmann, 1972). The scores were obtained by interviewing sixty students (twenty each from kindergarten, second, and fourth grade).

As of this date, the analysis is not complete. Therefore, it is inappropriate at this time to discuss results and conclusions. It should be noted that this study is a direct extension of a study presented at the 1975 NCTM Annual Meeting (Lamb & Steffe, 1975). The study makes an attempt to further refine techniques and provide insights into children's thinking about measurement.

References

Carpenter, T. P. The performance of first and second grade children on liquid conservation and measurement problems employing equivalence and order relations. An experimental report based on a doctoral dissertation from the University of Wisconsin, 1971.

Kundert, K. A. and Bargmann, R. E. Tools of analysis for pattern recognition. (Themis Report No. 22, Tech. Rep. No. 89.) Athens, Georgia: University of Georgia, 1972.

Lamb, C. E. and Steffe, L. P. Reasoning ability of chilren in the context of goal objects and logical implications. A paper presented at the meeting of the National Council of Teachers of Mathematics, Denver, April 1975.

Osborne, A. R. Metrication, measure, and mathematics. A Metric Handbook for Teachers, Reston, Virginia: National Council of Teachers of Mathematics, 1974.

A DESCRIPTION OF THE DIFFICULTIES SELECTED EIGHT YEAR OLD CHILDREN HAVE IN LEARNING THE DISTRIBUTIVE ALGORITHM FOR DIVISION

Linnea Weiland
Kean College of New Jersey

Robert Madell
Village Community School, New York, NY

Purpose

Despite the proliferation of research, a method of teaching the division algorithm effectively to all children has not been established. Although methods comparison studies (Van Engen and Gibb, 1956; Kratzer, 1972) have provided some guidance to the teacher, more exploratory research is needed to investigate the nature of children's thinking during the process of instruction. The results of an examination of students thinking during instruction may be used as a way of generating instructional reform (Menchinskaya, 1969; Brownell, 1972; McKeen and Eisenberg, 1973).

It is the purpose of this study to begin to describe how selected eight year old children who have had previous experience solving partitive division problems with base 10 materials learn during specified meaningful instruction in the distributive algorithm. In particular, our concern focuses on the difficulties encountered as children progress from the manipulation of concrete materials in support of their own reasoning to the use of symbols to support a standard argument.

Procedure

Subjects: The subjects for the study were sixteen eight year olds attending an independent elementary school in New York City. Prior to this study these children passed an initial qualifying task in numeration and then, individually, participated in eighteen weekly sessions in which they used base 10 blocks to solve partitive division story problems with two and three digit dividends and one digit divisors. During those sessions the children were given no specific instruction in division but were simply encouraged to solve the problems by whatever methods they could. All of the children demonstrated adequate procedures for using the materials to solve the problems.

Procedures: The investigators met each subject individually four times in a period of 3 weeks with each session lasting approximately fifteen minutes. All sessions were tape recorded. The children were taught to solve division examples (such as $3\overline{)795}$) by following the reasoning of the distributive algorithm (Kratzer, 1972). They were taught to record their work from left to right in the standard way but were not instructed in procedures for recording exchanges. The investigators interacted with the

subjects as difficulties arose. All examples involved either a one or two digit divisor and a three or four digit dividend.

Lesson 1: The standard notation for a division example was defined as a short hand for partitive story problems. Each child was then told that in contrast to his previous work, such examples should now be solved by a specific argument. In solving the first three examples, the child was helped to express the distributive argument using base 10 blocks and was helped to record the results appropriately. There followed twenty-five examples in which the child was to use that argument to find and record the number of hundreds in the quotient and to report how many hundreds were left undistributed. For these problems the blocks were not available.

Lesson 2: In the first two examples, the child was again helped to find and record the quotient by using the blocks to express the specified argument. The blocks were then removed and there followed eight examples in which the child's attention was focused on only the 100's digit. The child was asked in the next eleven problems to find both the hundreds and the tens in the quotient.

Lesson 3: No blocks were available. The children successfully solved three examples for only the 100's digit, seven examples for both the hundreds and the tens digits, and five examples for all three digits.

Lesson 4: No blocks were available. The eleven examples in this lesson included for the first time, some with four digit dividends. The children were asked to find all of the digits of the quotient.

Results

Sixteen categories were developed to classify the approximately 400 errors made by the children. Almost 90% of the incorrect responses fell into one of these categories. Six percent of these categories accounted for about 75% of classifiable error.

The most frequent types of error involved exchange. The following example will be used to illustrate two of the most common difficulties. In $3\overline{)785}$ the hundred remaining after the distribution of six of the seven hundreds must be changed into tens to make eighteen tens for further distribution. Some children would treat the remaining hundred as a ten and proceed to share nine tens among 3. Others would attempt to share ninety tens among three. These errors were made eighty nine times in similar situations by fourteen of the sixteen children in the study.

Thirteen children made seventy one errors by confusing the partial quotient with either the remainder or the division. Thus in an example such as $3\overline{)765}$, the child would record either one (the remainder in $7 \div 3$) or three (the divisor) in the hundreds place of the quotient.

In examples such as $4\overline{)236}$ children would frequently begin by reversing the divisor and the dividend and thus record two in the hundreds place of the quotient (4 ÷ 2 = 2). All but three children made a total of seventy such errors.

The sixth most common error (36 by 14 children) occurred when children neglected to exchange remainders. Thus in $4\overline{)222}$ the child would simply record 0 in both the hundreds and tens place of the quotient ignoring the two hundreds that properly should be exchanged for tens.

Recommendations

Although no formal statistical procedures are appropriate in a study of this sort, it is clear from the findings that most errors made by children are not idiosyncratic. The number of children making each of the various kinds of errors implies a commonality in children's thinking processes. Teachers concerned with meaningful learning need to be able to recognize these thought processes as a first step towards improving the quality of instruction. Further research of this sort is needed to clarify the nature of children's thinking during instruction.

References

Brownell, W. A. Problem solving. In J. Fred Weaver and Jeremy Kilpatrick (Eds.), The Place of Meaning in Mathematics Instruction: Selected Theoretical Papers of William A. Brownell. Vol. 21, School Mathematics Study Group, 1972.

Davidson, N.; McKeen, R.; and Eisenberg, T. Curriculum construction with student input. The Mathematics Teacher, 1973, 66, 271-275.

Kratzer, R. O. A comparison of initially teaching division employing the distributive and Greenwood algorithm with the aid of a manipulative material. Doctoral dissertation, New York University. Ann Arbor, Michigan, University Microfilms No. 72-11464, 1972.

Menchinskaya, N. A. Fifty years of Soviet instructional psychology. In J. Kilpatrick and I Wirszup (Eds.), Soviet Studies in the Psychology of Learning and Teaching Mathematics. Vol. 1. School Mathematics Study Group, 1969.

Van Engen, H. and Gibb, E. G. General Mental Functions Associated With Division. Cedar Falls, Iowa: Iowa State Teachers College Press, 1956.

THE EFFECTS OF PRECISE THINKING ALGORITHMIC EXERCISES ON THE FIFTH GRADE STUDENT'S ABILITY TO PERFORM THE DIVISION ALGORITHM

Gilberto Jose Cuevas
University of Miami

Purpose

This study was designed to investigate the effects of training in precise algorithmic exercises on the student's ability to successfully perform the division algorithm. The algorithmic activities in question, which were partially developed by Austin and Jesson at Sheffield University, England, and labeled Precise Thinking (PT) exercises, are based on concepts derived from the theory of finite-state and Turing machines.

Difficulties in the division of whole numbers may stem from the student's inability to follow the division algorithm or from incomplete knowledge of the multiplication algorithm. One possible way to help students overcome this difficulty is by involving them in a series of exercises with the objective of exposing the children to think in an algorithmic manner by following prescribed rules. Precise Thinking (PT) exercises are designed with this objective in mind. From a psychological standpoint the behaviors ellicited by PT exercises may be explained in terms of rule-governed behavior, which can be defined as behavior generated by a common algorithmic or generative procedure. PT exercises are activity-oriented. Theoretical discussions by Piaget, Brunner and Dienes indicate that activity-oriented instruction is more effective than more conventional types of instruction in terms of original learning, transfer and retention. Most current studies in mathematics education dealing with some aspect of division of whole numbers consist of the comparison of two or more methods for presenting the division algorithm. None of these studies were designed to determine if training in abstract algorithmic procedures has any effect on the student's ability to perform the division of whole numbers. The PT exercises require the students to select from a list of rules the specific rule which pertains to the pairing of given symbols, a behavior similar in nature to the operational definition of field independence/dependence. Research indicates that this latter construct is significantly related to mathematical ability, achievement and the ability to reason inductively.

Procedure

From three randomly selected schools in southeast Florida a group of 285 fifth grade students were administered Witkin's (et.al.), Group Embedded Figures Test and a division test. The sample participating in the experiment consisted of 33 field independent and 33 field dependent fifth grade pupils randomly selected from students found to be deficient in division of whole numbers. These students were given the How I Feel About Mathematics attitude scale, and they were then randomly assigned to one of three

treatment groups. The experimental treatment consisted of five half-hour sessions in PT exercises. The PT-rote group received the same number of sessions, with the students only required to memorize the rules used in PT. A third group remained in the classroom while the two previous groups were involved in the activities. The treatment period was followed by four 30-minutes sessions in which the "standard division algorithm" was presented to all groups. The How I Feel About Mathematics scale was then re-administered to the subjects along with a division posttest.

Results

The design for the study was a 2 x 3 completely crossed fixed-effects factorial design, with treatment (PT, PT-rote, base-line) and cognitive style (field independence/dependence) as independent variables. The dependent variables were performance on division and attitude toward mathematics. The attitude pretest scores served as covariate.

Analysis of variance and covariance indicated that: (1) participation in the PT exercises enabled field independent students to outperform all other pupils in the division algorithm, (2) the PT-rote group showed a less positive attitude toward mathematics than either the PT or base-line groups.

Within the limitations of the study, it can be concluded that PT exercises had a facilitating effect on performance on the division algorithm, but only for field independent students. Also, the PT activities promoted a more positive attitude toward mathematics than the rote-structured exercises.

Recommendations

The significance of the study lies in the fact that PT exercises, shown to facilitate division of whole numbers for a certain type of student, may suggest the use of abstract algorithmic procedures to ease the learning of other algorithms used in elementary school mathematics. Since the logical structure underlying PT is also the basis for computer operations, experience in PT exercises may have some positive transfer value to the understanding and manipulation of computers.

IMPLEMENTATION OF AN INDIVIDUALIZED APPROACH WITH COLLEGE FRESHMAN MATHEMATICS SURVEY COURSES: A STUDY

Thomas A. Egan
West Chester State College

John J. Kerrigan
West Chester State College

Mark F. Wiener
West Chester State College

C. Ralph Verno
West Chester State College

Purpose

The major purpose of the studies was to determine whether differences exist in student achievement and in attitude toward mathematics when college freshman mathematics survey courses are taught to students by an individualized instruction method and by a traditional instructional method. The studies also attempted to ascertain which type student performed best under the examined instructional modes.

Within the reported studies the method of individualizing instruction is called ILS (Individualized Learning Systems). It is supplemented by a computer managed instructional system called CMI. ILS/CMI is a systems approach to education, where, after proper diagnosis, students are allowed to proceed with their studies at their own rate of progress on material relevant to their learning needs. The ILS/CMI model has developed over the last decade at secondary and post secondary schools.*

Procedure

Two freshman mathematics survey courses were studied. The first, Mat 103, is a course required by the College of majors, other than elementary education. For three credits students must successfully complete four of the five Math Pacs; Logic, Probability, Statistics, Computing or Affine Geometry. The second, Mat 101, is the first course required of students majoring in elementary education. Its syllabus (Logic, Sets, Whole Numbers, Integers, Numeration Systems, Rational Numbers and Elementary Number Theory) was derived from a content survey of the mathematics texts most commonly used in today's elementary schools.

---

*The ILS model has been described by the authors at NCTM for Workshops in Philadelphia, PA., March, 1972; Springfield, Mass., November, 1973; Washington, D.C., April, 1975.

Students within these courses were studied in the Fall of 1973, Spring of 1974 and the Fall of 1974. In the Fall of 1973 Mat 103 students were randomly assigned to each of four sections - two ILS and two control. This was repeated for the Mat 101 students. Each involved professor (4) taught the same mathematical topics to one section each of traditional/lecture and ILS. Effort was exerted to guarantee that the only differences among the classes were those due to instructional methods. Students under ILS were allowed almost total freedom as to rate of progress, amount of retesting, time spent studying, etc. Only those students who completed all six instruments were used in the studies.

Students were administered a modification of the Aiken and Dreger Mathematics Attitude Scale and comparable department achievement examinations three times during the semester (pre, mid, post).

In the subsequent studies the procedures were varied by having other instructors, ones experienced only in the traditional courses, serve as controls; working with students who selected as well as those who were randomly assigned to the ILS approach; and the gradual evolution of the ILS/CMI method to one in which the students "earned" their freedom.

Based upon information provided at the time students were admitted to the College the Fall 1974 Mat 101 students were divided into three groups of approximately equal size, based upon their predicted first semester grade point average. These ability groupings (High, Middle, Low) were then examined to discover relationships among ability groupings, attitude and cognitive achievement.

Analysis

The basic statistical model was an AXB analysis of variance with repeated measures on factor B. Factor A contained two levels of instructional methodology (traditional, ILS) and factor B contained three levels of test administration (pre, mid, post). Simple effects were examined, using one-way analysis of variance or covariance when factor A or the interaction was found to be statistically significant. These were followed by Scheffé post hoc tests, where applicable. Trend analyses, both linear and quadratic, were performed when factor B was found to be statistically significant.

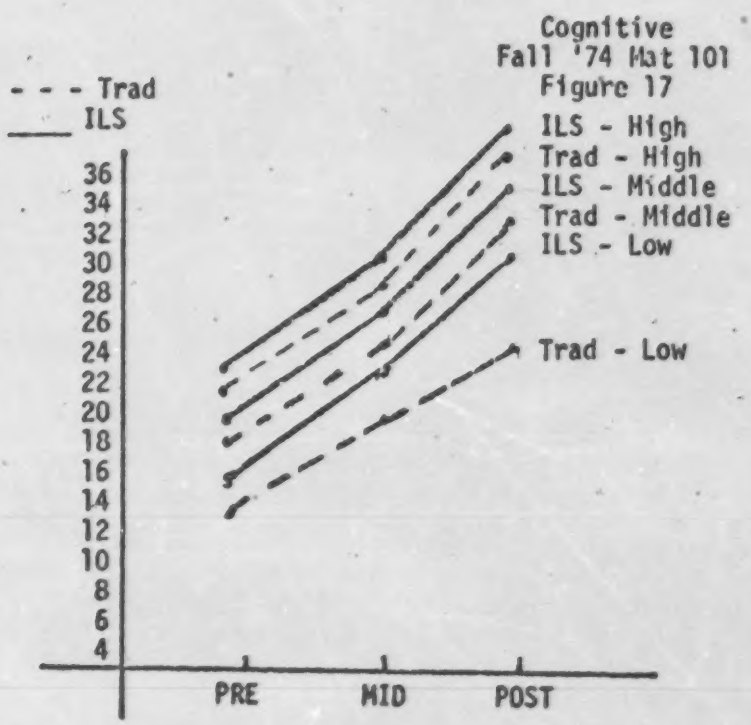
Results

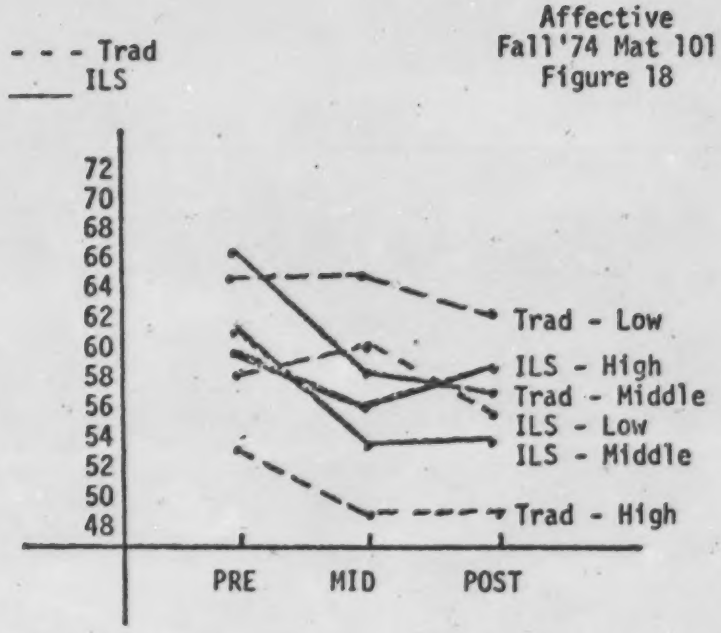
Fall '73: In Mat 103 both groups experienced learning ($F = 338.23$) with a significant method by trials interaction ($F = 4.94$). It was determined that the ILS group was superior cognitively to the traditional group by mid-term ($t = 3.05$) in spite of the fact that the traditional group was superior at the start of the project ($t = 3.29$). In Mat 101 both groups learned ($F = 450.5$), the interaction was significant ($F = 10.45$) and an analysis of covariance, on the adjusted post test scores, showed the traditional group superior to the ILS group at the conclusion ($F = 12.05$).

ILS Mat 101 students showed a more positive attitude toward mathematics ($F = 15.62$) during the study, but no affective differences existed in Mat 103.

Spring '74: ILS Mat 103 produced superior cognitive results ($F = 32.18$) across all trials. The interaction was significant ($F = 24.60$) and it was determined (ANACOVA) that the ILS students were superior at the semester's conclusion. Similar statements were made for affective results. In Mat 101, both groups learned ($F = 329.91$) and demonstrated an increasingly more positive attitude ($F = 17.47$) toward mathematics; however, no method demonstrated superior results.

Fall '74: No method produced significant changes in attitude among the Mat 103 students, but the ILS approach produces superior cognitive results ($F = 8.50$, $F_{AXB} = 153.24$), at the mid ($t = 4.30$) and post ($F = 6.99$) tests. In Mat 101 both methods produced learning ($F = 341.25$) but the ILS method produced superior results ($F = 9.81$) over all trials. Attitudinal results for Mat 101 favored the ILS approach ($F = 18.4$).





When the Mat 101 students' affective and cognitive results were examined on the basis of student ability levels it was found that the significant differences in methodology in favor of the ILS approach had occurred mainly within the lower ability students (Figures 17 and 18).

Recommendations

These studies suggest that the ILS/CMI approach produces cognitive results at least as good as those produced by traditional classes with the wide range of student abilities and attitudes typically found in survey courses of college freshman mathematics, and with at least two advantages. First, participating students and instructors demonstrate attitudes which may favor the ILS over traditional classroom procedures. Secondly, low ability students may be able to achieve better under an ILS approach than by a traditional approach. Additionally, these low ability students may exhibit a feeling of success as demonstrated by their expressing a more positive attitude toward mathematics than do their traditional counter parts. It is also evident that an individualized approach in which students must "earn" freedom by maintaining a learning rate comparable to traditional classes produces superior cognitive results to an approach in which students are granted carte blanche freedom as to pace of learning.

RESEARCH REPORTING SECTION #4

Presider: J. FRED WEAVER, University of Wisconsin
Madison, Wisconsin

Reports: READABILITY OF ACHIEVEMENT TESTS

JOE DAN AUSTIN, Emory University
Atlanta, Georgia

MARY ANN BYRNE, University of Georgia
Athens, Georgia

THE EFFECTS OF VOCABULARY AND TRANSLATION TRAINING ON SOLVING VERBAL PROBLEMS

GERALD KULM, Purdue University
West Lafayette, Indiana

MICHAEL WHEAT, Purdue University
West Lafayette, Indiana

EFFECTIVENESS OF AN INNOVATIVE CURRICULUM IN MATHEMATICS ON ATTITUDE TOWARD MATHEMATICS, SELF-CONCEPT, AND ACHIEVEMENT IN MATHEMATICS

DOROTHEA DORA SMITH, Alabama A and M University
Normal, Alabama

ATTITUDES CONCERNING THE NATURE OF MATHEMATICS OF SECONDARY SCHOOL STUDENTS

ROBERT R. O'REILLY, University of Ottawa
Ottawa, Canada

READABILITY OF ACHIEVEMENT TESTS

Dr. Joe Dan Austin
Emory University

Dr. Mary Ann Byrne
University of Georgia

Purpose

The relationship of predicted readability of achievement tests to grade level and cognitive achievement level is studied in this report. All fourth, seventh, eighth, and eleventh grade achievement tests of the National Longitudinal Study of Mathematical Abilities (NLSMA) were used to generate the test item pool. In the NLSMA tests all items were classified as computation, comprehension, application, or analysis (see Romberg and Wilson, 1969, pp. 41-43).

The Readability Formula II developed by Kane, Byrne, and Hater (1974) was used to compute predicted readability for achievement levels in each grade. For a sample of 400 tokens (i.e., words, mathematical symbols, or digits), the predicted readability is:

Predicted Readability = $-0.15A + 0.10B - 0.42C - 0.17D + 35.52$ where A is the number of "unfamiliar" words, B is the number of changes from word to mathematical token or vice-versa, C is the number of different "unfamiliar" mathematical symbols, and D is the number of question marks. A high predicted readability denotes a passage that is easy to read. (See Kane, Byrne, and Hater, 1974, for a more detailed discussion of this formula. Hater and Kane, 1975, discuss the development of this formula.)

Procedure

Samples of 400 tokens were obtained for each achievement level by counting from the first item in a given achievement level. When 400 tokens had been counted, the next 400 token sample count began with the next item. This procedure was repeated until no more complete samples could be formed. Samples of fewer than 400 tokens generally were not used. However, for fourth and seventh grade analysis there were not enough items for complete samples. Only here were partial samples used. For the predicted readability the observed readability variables were multiplied by the constant 400 divided by the actual number of tokens. This constant was 1.33 (302 tokens) for fourth grade analysis and 2.63 (149 tokens) for seventh grade analysis.

Analysis

The data was analyzed by a two way analysis of variance. The two factors were grade level and achievement level. The NLSMA tests had no eighth grade application items and no eleventh grade computation items.

It was necessary to estimate these cell averages. Using the procedure that minimizes the interaction effect, estimates were computed. (This procedure is referenced in Winer, 1971, p. 488 and computational formula given in Bennett and Franklin, 1954, pp. 382-383.) In the resulting analysis of variance the interaction degrees of freedom is reduced by two as two observations were estimated. The mean square and F-value is recomputed using this reduced degress of freedom. In all analysis the α-level used was 5%.

Results

For predicted readability the interaction and achievement level were not significant but the grade level was significant. As two cell averages were estimated and two cells had partial 400 token samples the conservative Scheffé (1959) method was used to compare the predicted readability for the grade levels. The resulting ordering ($\alpha = .05$) was:

(1) 4th grade $\geq$ 7th grade $\geq$ 8th grade $>$ 11th grade with the averages ordered as written in (1). Using the actual data without the two estimated averages, a linear regression of predicted readability on grade level was run. The resulting equation was:

Predicted Readability = 42.987 - 0.989 x Grade with an R-Squared of 0.511. A goodness of fit test gave $F(2,51) = 1.860$ so the lack-of-fit null hypothesis was not rejected.

The number of questions in the 400 token samples was also analyzed with a two way analysis of variance. This variable relates inversely to test item length. For example if 3 items are required for one 400 token sample and 6 items are required for another 400 token segment, the first three items are on the average twice as long as the second six items as measured by token length. The results of the analysis of variance for this variable was that the interaction was not significant but both grade level and achievement level were significant. The main effect averages were again analyzed with the Scheffé method. The statistical ordering ($\alpha = .05$) for grade levels was

(4) 4th grade $\geq$ 7th grade $\geq$ 11th grade $\geq$ 8th grade

with the lines indicating homogeneous subsets. For achievement levels the ordering was

(5) Computation $>$ Comprehension $\geq$ Analysis $\geq$ Application

Conclusions

The readability formula used in this study was developed for use with written mathematical text material. This formula does give a predicted readability for achievement tests that is significantly related to grade level but

not to achievement level. The linear relationship found between predicted readability and grade level needs to be replicated before it is accepted for use in setting any criteria bounds for achievement test readability. If there is an important relationship between readability and grade level on achievement tests - as this study indicates - standardized tests perhaps should also be analyzed with respect to their readability.

The results of the length of test items are as expected. In general the results indicate that test items tend to be longer for higher grade levels. However, the results also indicate that test item length is strongly related to achievement levels of the test items. This relationship needs to be further studied to investigate what implications it has for cognitive test item construction and analysis.

References

Bennet, Carl A., and Franklin, Norman L. Statistical Analysis in Chemistry and the Chemical Industry. New York: John Wiley, 1954.

Hater, Mary A., and Kane, Robert B. The cloze procedure as a measure of mathematical English. Journal for Research in Mathematics Education, 1975, 6, 121-127.

Kane, Robert B.; Byrne, Mary Ann; and Hater, Mary Ann. Helping Children Read Mathematics. New York: American Book Company, 1974.

Romberg, Thomas A., and Wilson, James W. NLSMA Reports No. 7 - The Development of Tests. Stanford, California: School Mathematics Study Group, 1969.

Scheffé, H. The Analysis of Variance. New York: John Wiley, 1959.

Winer, B. J. Statistical Principles in Experimental Design. New York: McGraw Hill, 1971.

THE EFFECTS OF VOCABULARY AND TRANSLATION TRAINING ON SOLVING VERBAL PROBLEMS

Michael Wheat
Purdue University

Gerald Kulm
Purdue University

Purpose

The purpose of the study was to train eighth grade students to recognize words and phrases in verbal problems and translate them to symbolic arithmetic operations. In addition, students studied the meanings of math vocabulary words which commonly occur in verbal problems. The effectiveness of the training was investigated by assessing the students' ability to solve verbal problems.

The appropriateness of this type of training for beginning problem solvers is important to assess. Instruction in problem solving is often very general (e.g., what are the data, the conditions, the unknown) or very specific (e.g., translate directly from words to equations a la Dahmus). A compromise between these approaches is represented in the present study. Specific skills in translation and vocabulary are developed without restricting the student to a mechanical procedure which applies only to a narrow class of problem types.

One of the greatest difficulties for students in comprehending written mathematical materials is in dealing with the variety of ways in which a symbolic expression may be expressed. This difficulty is less severe in translating from symbols to words, since any one of the alternatives are usually acceptable.

Example:

$n - 5$	n minus 5 5 less than n 5 subtracted from n decrease n by 5 n take away 5 The difference of n and 5

However, in translating from words to symbols, as in verbal problems, the difficulty is great. The student must be able to recognize and comprehend any of the alternatives and symbolize it properly. In the case of subtraction, the order of the minuend and subtrahend is different from one verbal version to another, but the order of the symbols is fixed. Similar difficulties arise in translating other operations.

In order to translate, students must first know the meanings of mathematics words. In a study of the familiarity of mathematical words, Byrne (1971) found the following percentages for seventh and eighth graders

who were familiar with these terms: decrease (70%), quantity (60%), constant (40%), exceed (20%), rate (60%), and quotient (70%). These and other "simple" words must be understood by the beginning problem solver. Comprehension of the relationships and conditions stated in verbal problems is severely restricted if a student must struggle with individual words and their specialized meanings in mathematics.

Procedure

Materials: Two sets of self-instructional materials were prepared by the experimenters. One set was intended to help students in Translation from words to symbols. The translation material contained five parts: addition and subtraction, multiplication and division, equals, combined operations, and hidden operations. Each part contained examples of words and phrases which translated to arithmetic operation and relational symbols, then practice examples were provided.

The second set of materials were written for Vocabulary study. Verbal problems were analyzed and a list of common words was generated. Next, those words with familiarity ratings (Byrne) less than 80% were chosen for study. Five lessons with ten words each were prepared. Definitions and examples of usage were provided for each word, then matching and completion exercises were provided for practice.

Method: The subjects were three eighth grade classes taught by one teacher in a rural school. Each class was randomly divided into four groups: Translation, Vocabulary, Translation and Vocabulary, and Control. The students studied the materials three days a week for two weeks under the supervision of their regular teacher. The Translation, Vocabulary, and Control groups were given unrelated self-study material to equalize the amount of time taken to cover the material. The classes had not studied problem-solving previously during the year, and the teacher provided no additional instruction on problem solving during the two week period.

At the end of two weeks, a ten problem posttest was given. In addition, a test of attitudes toward mathematics and problem-solving was given before and after the two week treatment. Finally, three weeks after the posttest, a ten problem retention test was administered.

Analysis

Since mathematics ability by treatment interactions were expected to be of interest, the students' previous semester's math grade was used to divide Ss into high and low ability groups. The design for analysis of the dependent measures was, therefore, a 2 (Vocabulary) x 2 (Translation) x 2 (Ability) x 2 (Test Occasion) analysis of variance with repeated measures on the last factor.

Results and Conclusions

The data are in the process of being analyzed, so results are not yet available. Results and conclusions will be reported in the paper read at the meeting.

EFFECTIVENESS OF AN INNOVATIVE CURRICULUM IN MATHEMATICS ON ATTITUDE TOWARD MATHEMATICS, SELF-CONCEPT, AND ACHIEVEMENT IN MATHEMATICS

Dorothea Dora Smith
Alabama A and M University

Purpose

The present study was designed to investigate the effectiveness of an innovative curriculum in mathematics on the black student's attitude toward mathematics, his self-concept, and his achievement in mathematics at the freshman level in thirteen predominantly black colleges.

A survey of the literature pertinent to this study revealed that there was very little organized knowledge about the characteristics of blacks enrolled in the predominantly black institutions of higher education. The introduction of such variables as attitude toward particular subjects and self-concept, at the college level, as significant variables in the investigation of relationships involving academic achievement in the predominantly black institutions could provide data about such students. A major problem facing educational planners in the black colleges has been the lack of adequate information about the black students who enroll in black colleges upon which to base program efforts. The pressing need is for some method of upgrading the entering black freshmen so that they can perform at "acceptable" levels. A vital factor in such method will be a determination of what factors in the black freshman's preparation are related to his failure to achieve satisfactorily. The present study examined and described the effectiveness of an innovative curriculum in mathematics, at the college level, on attitudes toward mathematics, self-concept, and achievement in mathematics on four generations of entering students in thirteen predominantly black colleges and universities.

In recent years, there has been a growing body of research investigations dealing with various facets of learning in higher education. Many of these investigations have been focused on selected non-intellective variables and their relationship to achievement in mathematics. Results from some of the investigations indicated that favorable attitudes toward mathematics and self-concept were positively correlated with high achievement in mathematics. Studies by Erickson (1970), Whipkey (1969), and Wickes (1967) showed some evidence that attitudes toward mathematics do change in a positive direction after exposure to selected innovative academic settings. There has been, also, related research which has examined the relationship between self-concept and academic achievement (e.g., Brookover et.al., 1962; 1965). Strawitz (1969) reported that the utilization of innovative materials and methods in mathematics did not produce any changes in self-concept of the subjects in his study.

Procedure

In the present study, attitude toward mathematics and self-concept were assessed through the use of seven items selected from the ISE Student Assessment Inventory (Blake and Morgan, 1967). Three of the items were used to assess attitude toward mathematics and four were used to assess self-concept. The seven items were tested for reliability, for each generation, through the use of Cronbach's Alpha Coefficient. The ACT Mathematics Test was used to measure achievement in mathematics. The t-test, for independent samples, was used to test the results of the pre-test scores for the four generations combined on each of the dependent variables studied. The post-test scores were subjected to analysis of convariance for individual generations, and the combined scores for the four generations were subjected, for analysis, to the techniques of factorial analysis of convariances. The criterion of significance for all tests of significance was the 0.05 criterion of significance. All pre-test results were used as convariates for the post-test analyses.

Results

The pre-test Hypothesis I-A (the hypothesis on attitude toward mathematics) was not upheld since difference in the means of the two groups was in favor of the group designated as the non-ISE* group. The pre-test Hypothesis I-B (the hypothesis of self-concept) was not upheld since there was no significant difference in the means of the two groups (ISE and non-ISE). The pre-test Hypothesis I-C (the hypothesis on achievement in mathematics) was not upheld since the difference in the means of the two groups was in favor of the non-ISE group. The posttest Hypotheses II-A and III-A (the hypotheses that the ISE group would show a more positive attitude toward mathematics than would the non-ISE group at the end of the freshman and sophomore years, respectively) were upheld. Hypothesis IV-A was not upheld for Generation I at the end of the senior year. The post-test Hypotheses II-B and III-B (the hypotheses that the ISE group would show a more positive self-concept than would the non-ISE group at the end of the freshman and sophomore years, respectively) were upheld since the difference in the means of the two groups was in the direction hypothesized. Hypothesis IV-B was not upheld for Generation I at the end of the senior year. Hypotheses II-C and III-C (the hypotheses that the ISE group would achieve higher in mathematics at the end of the freshman and sophomore years, respectively) were not upheld since the difference in the means of the ISE group and non-ISE group was not in the direction hypothesized.

Recommendations

The results of this investigation showed that attitude toward mathematics and one's self-concept do change in a positive direction after exposure to an innovative curriculum in mathematics. It seems safe to conclude that the ISE materials and methods were somewhat effective in

*ISE - Institute for Services to Education

producing the initial changes in these variables. Although there were some gains in achievement in the ISE group, the non-ISE group scored significantly higher than the ISE group at the end of the freshman and sophomore years, respectively. The investigator feels safe in concluding that the ISE materials and methods were not effective means in producing the gains in achievement in mathematics.

In view of the fact that the study is an ex post facto study and that three major factors limit the validity of the conclusions stated above, the investigator recommends that the study be replicated under carefully controlled conditions in order to determine and/or substantiate her findings and conclusions.

ATTITUDES CONCERNING THE NATURE OF MATHEMATICS OF SECONDARY SCHOOL STUDENTS

Robert R. O'Reilly
University of Ottawa

> Many people regard mathematicians as a race apart, possessed of almost supernatural powers. While this is very flattering for successful mathematicians, it is very bad for those who... are attempting to learn the subject.
>
> - W. W. Sawyer, Mathematician's Delight, Penguin, 1943, p. 7.

Purpose

Although attitudes of both teachers and pupils are thought to be important factors in the learning of pupils, little research can be found on these topics. The typical study in this area is limited to studying the interests of students or their liking for the subject, the class or the teacher (Dutton, 1956 and 1968). Most reviews of research on the teaching of mathematics ignores this topic entirely, (Henderson, 1963; Rombey, 1969), although a recent exception is the review by Aiken (1970).

The purpose of this project is to examine the attitudes and beliefs of secondary school mathematics classes, specifically: (a) the methods teachers of mathematics employ, (b) the beliefs of students about the nature of mathematics, and (c) their beliefs about the importance and utility of mathematics. The scales used to measure these attitudes were developed by the International Project for the Evaluation of Educational Achievement (IEA). The scales are described by R. M. Wolf in the report edited by Husén (1967).

These attitudes will be studied in relation to student characteristics, classroom environment and student achievement. Data will be drawn from a major study which included these variables (O'Reilly and Garland, 1974). This analysis provides an extension and one of the few replications of the IEA study.

Although attitudes of students towards mathematics may influence their achievement, it is probable that the nature of the attitude-achievement relationship changes during the student's school career. Research by Anttonen (1968) and Dutton (1968) indicate that attitudes do change from the elementary to the senior secondary level. Neale (1969) suggested that there is a reciprocal influence between attitude and achievement.

Although the review by Aiken indicates inconsistent results of studies of the attitude-achievement relationship, most studies indicated a low correlation between the two variables (Aiken, 1970).

The data from the IEA study indicate no clear relationships between achievement in mathematics and the attitudes that mathematics is difficult or important. One sub-group (non-mathematics majors in final secondary year) yielded a correlation of .13 for the latter variable (Husén, Vol. II, 1967). Neale's analysis of the results of the American students who participated in the IEA study also showed no relationship (Neale, 1969).

A common assumption of many educators is that discovery, or heuristic, methods of instruction lead to higher levels of achievement.

A recent essay on this topic by Hughes (1974) traces this movement over the past century. He points out that heuristic techniques fall in the domain of mathematical thinking which is psychological rather than in the domain of mathematical thought which is logical.

Jerome Bruner, in an analysis of student thinking in mathematics, points out that young students must begin with sensory objects and then gradually reduce the specific properties until an abstract level is attained.

> However, once abstraction is achieved, the learner becomes free in a certain measure of the surface appearance of things, he nonetheless continues to rely upon the stock of imagery he has built en route to abstract mastery. (Bruner and Kenney, 1973, p. 436).

Thus the success of heuristics, for the student, is the stock of early concrete experiences and subsequent sets of imagery which permits the student to work at the level of heuristics.

The IEA report reports positive correlations between the view that teachers use "discovery techniques" and achievement for the sample of 13 year-olds, but small and negative correlations for 18 year-old students (Husén, Vol. II, p. 145-149). Husén suggests that the latter result may be the result of conversative methods of teaching or of the nature of the examination system (Husén, Vol. II, p. 299, 1967).

The value of replicating the test of this hypothesis at this time in the Province of Ontario is suggested by two facts. Modern mathematics programs, with emphasis on "discovery methods" were mandated by a central authority throughout the province beginning in 1962. Teachers in the Province are not thoroughly trained in the program; pupils have known only this type of program. Secondly, there are no important centrally-imposed examinations for the Ontario secondary school students who participated in this study. Thus Husén's explanations would not be applicable in this situation.

A surprising result of the IEA calls for a serious re-examination of the nature of mathematics as viewed by secondary students. Generally there was a negative correlation between achievement and the belief that mathematics is an open system. The association of scores was stronger for older students. Students who were most interested in mathematics tended to view it as a fixed, formal closed system.

Since these results are contrary to the expectations of most authors of modern mathematics programs, the hypothesis should be re-examined with a population which has had extensive experience in the modern curriculum.

Husén also found that the belief that mathematics is important is related to achievement (Vol. I, p. 113, and Vol. II, p. 148).

Procedure

The unit of analysis in this study is the classroom, and all scores are average classroom scores. Forty-eight classrooms from 12 secondary schools from four boards of education in Eastern Ontario constituted the sample. To fulfill certain requirements of the major project, a combination of random and judgmental sampling was employed.

The forty-eight classes were equally divided between the grades 9 and 10 levels (or equivalents) and between advanced academically-oriented and general or terminal classes. A total of 1100 students participated.

Data were collected from students with the following instruments:

1. Stanford Achievement Test (SAT): Mathematics Form W.
2. Biographical data:
    a) parents education,
    b) dislike for school,
    c) future planning.
3. Learning Environment Inventory (LEI).
4. School and College Ability (SCAT) Test, Series II.

Questionnaires and tests were administered in two sittings with each of the 48 classes. One sitting involved the administration of the Stanford Achievement Test-Mathematics - Part A, and the appropriate forms of the SCAT-Series II. The Stanford was administered to 2/3 of the class while the SCAT was administered to the remaining one third. In the other sitting, the LEI and the Biographical Inventory were administered to one half of the class while other tests were administered to the remainder of the class. Males and females were proportionally represented in each segment of data collection.

The order of administering sets of instruments in the two settings was alternated to eliminate order effects.

The Learning Environment Inventory (LEI) was developed by Walberg and Anderson for the evaluation of the Harvard Project Physics. The instrument consists of 105 items describing typical school classes. Each student expresses his agreement or disagreement on a four point scale. The items are grouped into 15 scales.

The intra-class correlations for class groups range from .31 to .92 for each scale. The LEI was successfully utilized in a variety of experimental and correlational studies to relate classroom climate to teaching methods and to achievement (Anderson and Walberg, 1974).

Results

Since attitudes was not a major variable of the later study only general summary data are now available. Detailed analysis of the attitude data is not under way. Statistical techniques to be employed include correlation, partial correlation and canonical analysis. Separate analyses will be computed for the total group and for various sub-groups.

Preliminary analysis indicates the following:

1. Students in the sample have similar, but consistently more favorable attitudes than U.S. students in the IEA project. Scores for the attitude that mathematics is an open process are notably higher.

2. Classroom climate as measured by the LEI is related to only one attitude scale: Views about mathematics teaching (multiple R=.75).

3. The relationship between attitudes and achievement have not yet been established.

4. One canonical variate has so far been identified (r_c=.92). The factor is cognitive in character. The greatest weight on the criterion side is achievement. The variate is characterized by attitudes that mathematics is relatively easy and relatively closed. On the predictor side, the factor is high on aptitude and on the LEI sub-scale apathy.

Further analyses and interpretations will be available at the time of the annual meeting of the NCTM.

References

Aiken, Lewis R. Attitudes toward mathematics. Review of Educational Research, 1970, 40, 551-596.

Anderson, Gary J. and Walberg, H. J. Learning environments, in H. J. Walberg (Ed.), Evaluating Educational Performance, Berkeley, California: McCutchan Publishing, 1974.

Anderson, Gary J. Effects of classroom social climate on individual learning. American Education Research Journal, 1970, 7, 135-152.

Anderson, Gary J. and Walberg, H. J. Classroom climate and group learning. International Journal of the Educational Sciences, 1968, 2, 175-180.

Anttonen, R. G. An examination into the stability of mathematics attitude and its relationship to mathematics achievement from elementary school to secondary school level. Doctoral dissertation, University of Michigan, 1967. Ann Arbor, Michigan: University Microfilms, 1968, No. 68-1521.

Bruner, Jerome S. and Kenney, Helen J. Representation and mathematics learning, in J. M. Anglin (Ed.), Jerome S. Bruner: Beyond the Information Given. New York: W. W. Norton, 1973.

Dutton, Wilbur H. Attitudes of junior high school pupils toward arithmetic. School Review, 1956, 64, 18-22.

---------. Another look at attitudes of junior high school pupils toward arithmetic. Elementary School Journal, 1968, 68, 265-268.

Henderson, Kenneth B. Research on teaching secondary school mathematics, in N. L. Gage, (Ed.), Handbook of Research on Teaching. Chicago: Rand McNally, 1963.

Hughes, Barnabas B. Heuristic teaching in mathematics. Educational Studies in Mathematics, 1974, 5, 291-299.

Husén, Torsten (Ed.). International Study of Achievement in Mathematics: A Comparison of Twelve Countries. New York: John Wiley, two volumes, 1967.

Neale, Daniel C. The role of attitudes in learning mathematics. Arithmetic Teacher, 1969, 16, 631-640.

O'Reilly, Robert R. and Garland, Parnell. Paradigm for Evaluation in the High School. Ottawa: University of Ottawa, 1974.

Romberg, T. A. Current research in mathematics education. Review of Educational Research, 1969, 39, 473-492.

Sawyer, W. W. Mathematician's Delight. London: Pelican Books, 1943.

RESEARCH REPORTING SECTION #5

Presider: JOHN W. GREGORY, University of Florida
Gainesville, Florida

Reports: CONFERENCES OF THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS: PERCEPTIONS OF SELECTED PERSONNEL

JAMES D. GATES, National Council of Teachers of Mathematics
Reston, Virginia

AN ANALYSIS OF THE RELATIVE EFFECTIVENESS OF VARIOUS REMEDIAL MATHEMATICS PROGRAMS FINANCED BY TITLE I, ESEA FUNDS IN NEW YORK STATE.

GEORGE AMAN, New York State Education Department
Liverpool, New York

FIELD TESTING A SYSTEMS ANALYSIS MODEL FOR MATHEMATICS/SCIENCE TEACHER EDUCATION

MARGARET A. FARRELL, State University of New York at Albany
Albany, New York

WALTER A. FARMER, State University of New York at Albany
Albany, New York

RICHARD CLARK, State University of New York at Albany
Albany, New York

FUTURE AND PRESENT EFFECTIVENESS OF COMPUTER GRAPHICS

BERNARD BALTZ, The Ohio State University
Columbus, Ohio

CONFERENCES OF THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS: PERCEPTIONS OF SELECTED PERSONNEL

James D. Gates
National Council of Teachers of Mathematics

Purpose

The purposes of this study were: (1) to identify the expectations of school superintendents, supervisory personnel, and classroom teachers of secondary school mathematics for the attendance of teachers at in-service teacher education conferences in mathematics; (2) to ascertain the perceptions of school superintendents, supervisory personnel and classroom teachers of secondary school mathematics with respect to the attainment of their expectations; and (3) to be able to improve future conferences in order to better meet the needs of teachers of secondary school mathematics.

It is the belief of this researcher that the data collected and the information presented in this study will provide a basis for measuring the effectiveness of NCTM conferences and assist planners of future in-service conferences for teachers of secondary school mathematics.

Large sums of money are being spent each year on in-service teacher education conferences designed to bring about change, and presumably, improvement of instruction in our schools. For example, one of the most significant aspects of the professional activities conducted by the NCTM is its conference program. Ten such conferences were held during a recent school year attracting nearly twenty-two thousand persons interested in the teaching of mathematics at the elementary, secondary, two-year college, and teacher-preparatory college levels. But we know little today either about the effects of these conferences, or, more important, how to bring about comprehensive change in the schools. The NCTM is anxious to improve its conferences in order to better meet the needs of today's teachers and is interested in finding ways to bring about these improvements.

This study should help the profession to measure the success of in-service conferences with respect to the expectations of several concerned groups. If research justifies the many benefits and values claimed for such meetings, the teaching profession, our entire educational system, and society in general will benefit from the effective application of the conference method of learning.

Procedure

Five populations were considered for study with regard to this problem--school superintendents, supervisory personnel, classroom teachers, students, and parents. It is felt that each of these sets of people has expectations regarding the attendance of teachers at in-service conferences. The populations of students and parents were omitted from the study. In the opinion of this researcher, the students did not have any perceptions of expectations for conference attendance by their teachers. No satisfactory method is known by this researcher to ascertain the expectations of parents.

Questionnaires were sent to random samples of school superintendents, mathematics supervisors, and classroom teachers of mathematics who had attended NCTM conferences during 1971. The data gathered were analyzed relative to the expectations of these three populations. From this information it is anticipated that more will be learned about the effects of these in-service conferences, how to improve the conferences, and therefore how to better meet the needs of today's teachers of mathematics. One of the aspects of the survey which received special attention was the question as to how the three populations perceived the attainment of their expectations.

Results

The findings that follow are summarizing statements from the data presented in the study. These data were obtained from the replies to questionnaires sent to random samples of superintendents, mathematics supervisors, and conference registrants.

1. The results of this study show that the NCTM conference registrants had the following characteristics:

a) Nearly 82 percent were from urban or suburban communities,
b) The average age was forty-two,
c) More than 53 percent were male,
d) The average length of teaching experience was fourteen years,
e) Nearly 73 percent had earned at least a master's degree,
f) The average number of NCTM conferences attended was five.

2. The superintendents, supervisors, and conference registrants participating in this study seem to feel that the most important reasons for attending conferences of professional subject-matter associations are to learn about new instructional materials and upgrade instructional procedures. Most respondents indicated that these expectations were met through attendance at NCTM conferences.

3. The results of this study show that there is great interest in workshops on concrete materials, demonstration classes, and lectures on mathematical topics as types of sessions for future NCTM conferences.

4. The results of this study show that there is great interest in the following topics for inclusion on programs at future NCTM conferences: individualized instruction, providing for slow students, applications of mathematics, and behavioral objectives.

5. The results of this study show that some financial support was made available to two thirds of the registrants for attendance at NCTM conferences. Superintendents seem to be receptive to receiving requests for financial support for teacher attendance at future NCTM conferences. The reason given by most respondents for not giving or receiving reimbursement was that there was no money in the budget for this activity.

6. The results of this study show that it would be highly desirable for the NCTM to continue the practice of trying to have a conference annually within five hundred miles of all teachers of mathematics.

7. The results of this study show that there is no significant difference between the expectations, nor the perception of their attainment, on the part of respondents from different types of communities (urban, suburban, and rural or small town).

Conclusions

The generalizations that follow are supported by the findings from the data presented in the study and summarized above.

1. Conferences sponsored by professional subject-matter associations such as the NCTM are an effective way of meeting the needs and expectations of superintendents, supervisors, and classroom teachers with respect to in-service education of secondary school teachers of mathematics.

2. Future in-service conferences planned by professional subject-matter associations such as the NCTM should include much the same type of activities as are currently being offered.

3. Attendance on the part of large numbers of classroom teachers at in-service conferences planned by professional subject-matter associations such as the NCTM is dependent upon the ability of the school district to provide reimbursement for some or all expenses.

AN ANALYSIS OF THE RELATIVE EFFECTIVENESS OF VARIOUS REMEDIAL MATHEMATICS PROGRAMS FINANCED BY TITLE I, ESEA FUNDS IN NEW YORK STATE

George Aman
New York State Education Department

Purpose

Many school districts in New York State have conducted, and presently are conducting, a compensatory program in the area of mathematics. Many different teaching methodologies are used across districts. These may include the laboratory approach, programmed instruction, small group instruction, the use of tutors or computer assisted instruction.

Although each district must complete an evaluation of its own project at the end of the year there is little research which studies the relative effectiveness of such projects. No overall analysis of a large number of projects is done to determine whether a math lab approach is more effective at grade three than at grade six.

There are similar unanswered questions when one considers the relationship between the amount of deprivation in mathematics and the effectiveness of a particular type of program. It may be the case, for example, that a sixth grade child who is two or more years behind grade level, can be helped significantly by a program based on tutoring but that some child who is only one year behind may benefit more from a math lab approach.

Presently judgments about possible effectiveness are often made without evaluative research which would be very helpful in giving direction when choices must be made. Is a math laboratory approach the best type of program for junior high students? Will small group instruction work well with third graders? Do students who are more than two years below grade level benefit more from one-to-one tutorial programs than from other types of programs?

The purpose of this study is to make a step in the direction of judging the relative effectiveness of different instructional techniques used in compensatory mathematics programs that have been funded by Title I monies. These programs will have been conducted during the 1974-75 school year and shall include all districts in New York State outside of New York City that conducted Title I mathematics programs. It is hoped that with the results of this study, districts will be better able to make decisions about how to spend the limited Title I funds available for compensatory programs.

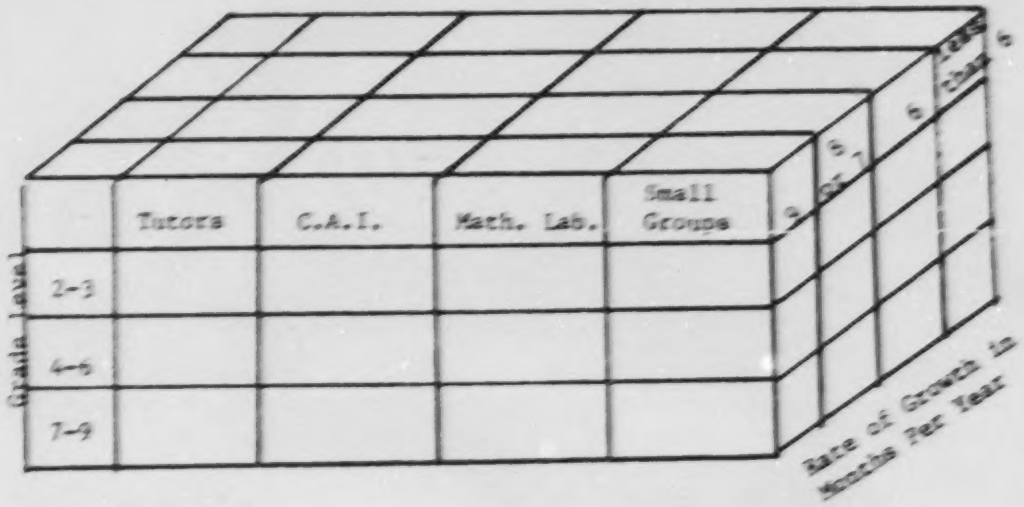
Although much work has been done in the evaluation of individual compensatory programs, there are no studies that directly relate to the questions raised in this study.

Procedure

Data will be collected in July, 1975, for four different types of mathematics programs. All will have been funded by Title I, ESEA, during 1974-75 and will have been conducted in grades two through nine. Each program will have been conducted for at least six months.

Analysis

A three way analysis of variance will be used to analyze the data. There will be three grade level groupings, four methods of instruction and four amounts of deprivation. The amounts of deprivation will be based on rate of growth in months per year. Also the dependent variable, gain scores, will be translated into so many months per year since treatments may vary from 6 to 10 months. A treatment subject will be defined as a group of students at one of the grade span designations. Thus the analysis of variance table will be a 3 x 4 x 4 table which looks like the following.



Since the data has not yet been received from the approximately 200 districts to be involved, the results and conclusions are not yet available. However, they should be available by the early fall of 1975.

FIELD TESTING A SYSTEMS ANALYSIS MODEL FOR MATHEMATICS/SCIENCE TEACHER EDUCATION

Margaret A. Farrell
State University of New York at Albany

Walter A. Farmer
State University of New York at Albany

Richard Clark
State University of New York at Albany

Purpose

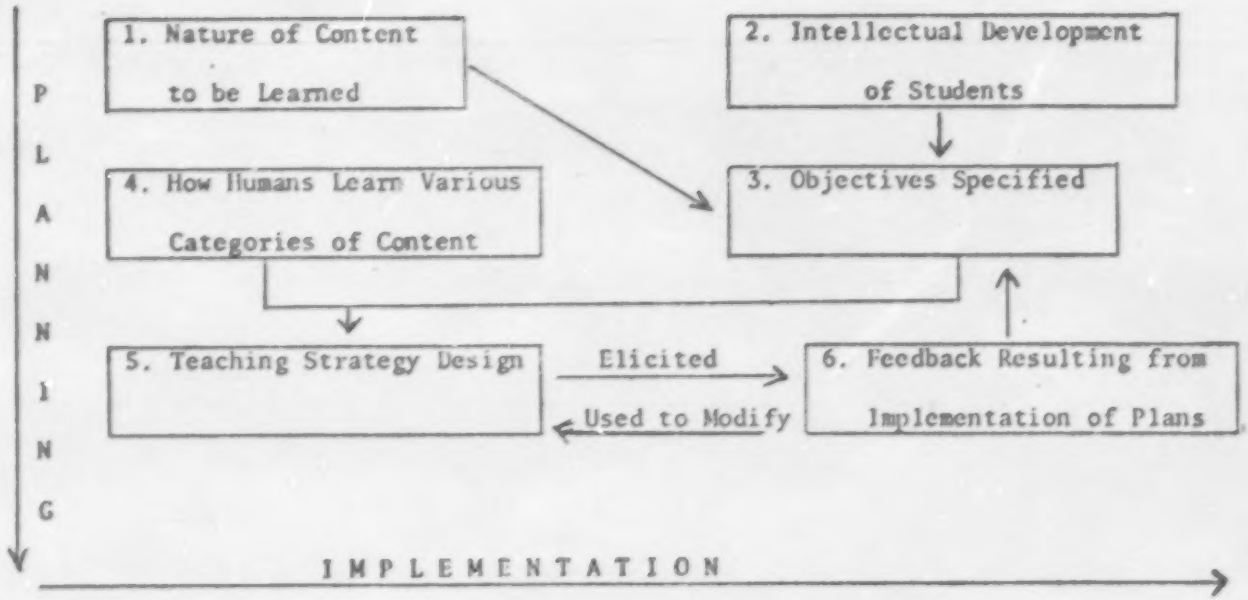
There were two primary purposes for conducting the research:

(1) to test a systems analysis model by gathering data on student teachers' performance of classroom teaching competencies having specific lines of correspondence to the model, and

(2) to discover inter-relationships among the separate competencies comprising the total instrument.

Currently there is much interest in competency based teacher education but empirical studies in this area, particularly in secondary school mathematics teacher education, are lacking. However, there have been some team efforts in teacher education--e.g., the work of Cooney et. al. based on the theories of Henderson. There have been implications for instruction in the works of Gagne, Ausubel, Piaget, and Bruner. Various national and regional groups (e.g., NCTM, AASTEC) have developed guidelines for identifying teacher competencies but, as yet, instruments tied closely to theoretical models and submitted to the test of data-gathering and analysis do not appear in the literature. Since a growing number of states are mandating CBTE, it seems important to conduct relevant research which ties together an identified theoretical basis for the approach and data on the results achieved. The results of such studies can provide a basis for the development of future teacher education programs which will be both practical and generalizable.

The systems analysis model depicted below is based on the author's prior experiences in supervising 500+ student teachers, the growing body of research evidence by cognitive psychologists such as Ausubel and Gagne and the developmental psychology of Piaget. This simplified model forms the conceptual framework for the Albany Mathematics Science Teaching Program (AMST), which the authors designed and co-directed since its implementation in the spring of 1973.


1. Nature of Content to be Learned
2. Intellectual Development of Students
3. Objectives Specified
4. How Humans Learn Various Categories of Content
5. Teaching Strategy Design
Elicited
Used to Modify
6. Feedback Resulting from Implementation of Plans
PLANNING
IMPLEMENTATION

Boxes 1, 2, 4 and 5 represent models within the overall systems analysis model. The model for the nature of content was derived from the work of national curriculum groups plus Max Bell's work on the structure and process of mathematics. The research of Piaget forms the basis for the box dealing with intellectual development. Gagne's hierarchies of types of human learning and the conditions which bring each about are coupled with Ausubel's work on advance organizers and meaningful verbal learning in box #4. These in turn find application, as does Piaget's work, in the teaching strategies designed to promote the acquisition of specific categories of learning tasks (i.e., concept vs. rule learning, etc.). The taxonomies of Bloom, Krathwohl and Harlow are included in the specification of objectives in all three domains. Finally, feedback giving and getting among components plays a central role in the model.

Procedure

Data were gathered over a period of five semesters from spring 1973 through spring 1975 on the classroom performance of 122 enrollees, of which 100 successfully completed the program. Each student teacher was observed by one of the researchers a minimum of eight-45 minute teaching periods during their ten weeks of student teaching. A similar number and kind of written observation were also compiled by the public school cooperating teacher to whom each student was assigned. Summative evaluation based on the data resulted in an assessment of one of three kinds: below criterion, meets criterion, or exceeds criterion. (Exceeds criterion was achieved by a very small sample of students.) In order to maximize inter-observor

reliability in data-gathering, the researchers frequently observed the same class session, privately recorded relevant data, and then cross-checked the written record. Likewise, the researchers regularly read data collected by each other and discussed the kind of inferences to be drawn as related to the AMST instrument.

After an n of 100 had successfully completed the program, tabulations of the students' performance in each item were compiled. Inter-correlations are being run on these data and clusters which appear will be examined in terms of their logical connections to the thought model which undergirds the program and the instrument.

Results

Some items on the competency list (AMST instrument) are almost universally performed at criterion level; others have proved to be of moderate difficulty; and several are achieved by less than half of the student teachers successful in this program. No data were compiled for unsuccessful student teachers since they are not allowed to continue in the program through the final weeks of student teaching.

Inter-correlations will be run during the summer and early fall of 1975. Hence results of this analysis are not yet available. However, visual inspection of the tabulations compiled to date indicates some obvious clustering of competencies which bear logical relationships to one another and to common components of the model. Thus it is expected that significant inter-relationships will be demonstrated when data analysis is completed.

It is expected that the data analysis will: (1) verify a number of epistemic correlations between thought model and specific competencies and clusters of competencies, and (2) reveal the inter-dependencies among specific competencies included in the AMST instrument.

FUTURE AND PRESENT EFFECTIVENESS OF COMPUTER GRAPHICS

Bernard Baltz
The Ohio State University

Purpose

The purpose of the study was to demonstrate that graphics sufficient to the pedagogical need can be generated quite easily, and that students can learn in the solitary environment of a console. One aspect of the computer revolution which holds great promise for mathematics teaching is the capability of program-controlled animated graphics. Many graphics systems/languages exist with this capability. In general, these can make points or lines on a cathode ray tube (CRT) using some coordinate system. The use of Fortran-like languages enables the user to specify locations, movements, intensities, rotations, and the like with calculated precision.

Mathematical objects, being easily coordinatized, can be easily drawn and manipulated. Already many computer animated mathematical films have been made both as supplements to textbooks and as short features which deal with subjects in an informal pictoral manner previously impossible (e.g., Space filling curves, homotopies in the plane).

On the other hand, advances have been made in designing CAI curricula, and in developing coordinated graphic capabilities (Best, 1971; Graham, 1971; Hammond, 1971; Hicks, 1971; McClintock, 1973; Rigney, 1974; Turner, 1974). However, as yet, no sophisticated interactive graphics systems have been coordinated with educational branching-type programs. The reason is that graphics terminals are still relatively expensive and that the appropriate software has not been fully developed. But given the extent of current research in graphics software and the general decline in hardware prices, it is quite reasonable to expect moderately priced graphics units, usable in a TSO (time sharing option) setting, in the near future.

The above-mentioned developments will mean TSO terminals with high quality and detailed diagrams on screen. Students will be able to enter numerical or physical input (by lightpen or other devices) and see accurate immediate results, with subsequent branching to appropriate visual examples and exercises.

It is not advocated that such computerization supplant traditional class-lecture education. However, in the case of remedial or advanced topics, or for any of a variety of "obscure" topics particular to some students' interest (for which the teacher does not have time or training), lessons of this type could be most helpful. They would be modules handled basically like coursewriter lessons are done today.

Given these potentialities, mathematics educators must confront several questions:

1. Would students learn as well or better (in terms of speed, transferral, retention) given visual (animated?) examples (diagrams, functions, distributions, etc.)?

2. In general, can CAI modules be designed (incorporating visual examples) which provide adequate branching?

3. Can (and should) there be a standard visual learning set, to integrate a visual approach?

4. Finally, is this approach worthwhile, desirable?

It is time to begin examining these questions from the somewhat objective standpoint of knowing roughly where the state of the computer art will leave us in a very few years. Perhaps a large number of independent modules will be written, or possibly ultimately large scale programming will be written in a way analogous to the SMSG writing teams. In any case the mathematics education community ought to begin probing the questions.

An important question to address is that of the effectiveness of computer generated visual materials. Many studies have attempted to determine the significant visual and auditory parameters in learning through media (Bourne et. al, 1972; Copper, 1972; Hoban, 1971; Kalkofen, 1972; Teather, 1975).

Procedure

Two pilot experiments and a main experiment were performed. The first tested the technical feasibility of the mode of presentation. A lesson concerning graphing trigonometric functions was made from an IBM 1130 and put onto videotape cassettes. Students' attitudes towards the particular mode of presentation along with their comparative performances were examined. No major problems were encountered.

The second pilot was a run-through of the main experiment with a small class, to test the experimental presentation for flaws and to polish the control presentation. Also intervening variables will be sought. (This pilot was performed Summer, 1975, with n=14.)

In the main experiment two groups are randomly selected from a course in geometry for elementary teachers. Three class lessons were produced on videotape for each of the two groups. The subject matter is rigid motions and symmetry groups of simple mathematical objects, of necessity a topic of visual significance. One set of tapes involved traditional visual aids and pedagogy. The experimental treatment involved extensive visual manipulations.

The cassettes were made from a graphics system adapted for both filming and real-time screen viewing (GRASS, Graphics symbiosis system, on a PDP11). The experimental treatment involves a minimum of verbal information and

student calculation. In the design of the content and organization of presentation, known results concerning relevance, repetition, iconicity, and interference were considered. The style of the graphics, however, was largely determined by the graphics language used.

Probably the most difficult factor to control in this experiment is the mode of question on the post-test instrument. As Rigney points out in his treatise, (1974) "Computer graphics might be more memorable than verbal statements at the level tested by recognition tests but this might not hold for tests of semantic processing." A scheme of rating test items along these lines was tried in the first pilot experiment and was found to be inadequate. The final instrument will be constructed with care, and if mode of question can be reasonably assigned, then interaction with treatment will be scrutinized.

The differences in performances will be examined using the appropriate standard tests, and analysis of covariance may be employed if the pilot warrants its use. Hopefully the computer visual manipulations will be shown to be a viable alternative to lecture and handwork in this instance.

References

Best, E. The development of visual materials for CAI. Technical Memo #5 ERIC Microfiche ED 072 625 (1971).

Bourne, L. E.; Dominowski, R. L.; and Ekstrand, B. R. The Psychology of Thinking, Prentice-Hall, 1971.

Coppen, H. Film Research in Great Britain. Educational Media International, 1972, #2.

Graham, A. and Read, G. A television program on finite differences. Educational Studies in Mathematics, 1971, 3(2), 206-219.

Hammond, K. R. Computer graphics as an aid to learning. Science, 1971, 172, 903-908.

Hicks, B. L. Cidac--a concept in educational technology. Audiovisual Instruction, 1971, 16(7), 68-70.

Hoban, C. F. The state of the art of instructional films. ERIC Microfiche No. ED 055 432, (1971).

Kalkofen, H. Instructional film research: some notes on methodology. Educational Media International, 1972, 2.

McClintock, P.A. and Kimberlin, D. A. An on-line electronics graphics symbol set for the Plato IV system. ERIC Microfiche No. ED 095 925, (1973).

Rigney, J. W. and others. Interactive computer graphics for performance-structure-oriented CAI. ERIC Microfiche No. ED 095 848.

Teather, D. C. B. and Marchant, H. Learning from film with particular reference to effects of cueing, questioning, and knowledge of results. Programmed Learning and Educational Technology, 1974, 11(6), 317.

Turner, R. C. PIRATS- a modified coursewriter plus graphics for the DEC PDP 11/20. ERIC Microfiche No. ED 099 003, (1974).

RESEARCH REPORTING SECTION #6

Presider: DOUGLAS K. BRUMBAUGH, Florida Technological University
Orlando, Florida

Reports: THE EFFECTS OF CALCULUS IN HIGH SCHOOL ON COLLEGE CALCULUS PERFORMANCE

DENNIS M. LUCIANO, Saint Joseph's College
Philadelphia, Pennsylvania

AGNES M. RASH, Saint Joseph's College
Philadelphia, Pennsylvania

AN INVESTIGATION OF THE RELATIONSHIPS BETWEEN SELECTED PERSONALITY CHARACTERISTICS OF EIGHTH-GRADE STUDENTS AND THEIR ACHIEVEMENT IN MATHEMATICS

DARYLE CLINE MAY, Jacksonville University
Jacksonville, Florida

THE EFFECT ON THE LENGTH OF THE NUMBER LINE AND THE NUMBER OF LINE SEGMENTS IN EACH UNIT SEGMENT ON SEVENTH-GRADE STUDENTS' ABILITY TO LOCATE PROPER FRACTIONS ON THE NUMBER LINE

CAROL F. NOVILLIS, Florida International University
Miami, Florida

ASPECTS OF TEACHER DISCOURSE AND STUDENT LEARNING OF A MATHEMATICAL CONCEPT

LYLE R. SMITH, Augusta College
Augusta, Georgia

JAMES H. ROLLINS, Texas A & M University
College Station, Texas

THE EFFECTS OF CALCULUS IN HIGH SCHOOL ON COLLEGE CALCULUS PERFORMANCE

Dennis M. Luciano
Saint Joseph's College, Philadelphia, PA

Agnes M. Rash
Saint Joseph's College, Philadelphia, PA

Purpose

One of the changes in the mathematics curriculum of the post-Sputnik era was the downward movement of topics to lower grade levels. The impetus given to science and technology led to much interest in curriculum development at the elementary and secondary levels. University faculty participated in the design of new curricula. By the early 1960's students could complete second year algebra and trigonometry as early as 10th grade, but most college-bound students took this course in 11th grade. (Prior to this change, advanced algebra, trigonometry, and often solid geometry were given in the 12th grade.) As a corollory, new senior year mathematics courses were needed. Inspired by the current interest in the abstract, some schools introduced courses called "modern mathematical analysis," probability and statistics, modern algebra, or a combination of these. In addition, many schools inaugurated calculus courses. A number of years have passed since the introduction of these programs and it is now time to evaluate their effects.

The purpose of this study was to assess the effect of a year (or more) of calculus in the high school on the performance of students in calculus in college. None of the other programs was isolated for study.

At our institution, calculus is a three semester sequence of courses, each of which is four credits. Since many students, particularly biology majors, terminate the calculus sequence after two semesters, our exposition concentrates on the first two semesters. Over the past four years, 32% of the calculus students who entered St. Joseph's College had at least a full year of calculus at the secondary level, 63% had various other four year programs, and 5% had only the equivalent of three years, through algebra II and trigonometry. The sample for the latter group was too small to make a comparison with the other groups. Of the students who had calculus, 14% entered as advanced placement and began college calculus with the second course. Only 1% received credit for two semesters of calculus. This leads us to believe that the primary purpose of calculus at the secondary level is not advanced placement. Conversations with students indicate that most of them do not seek acceleration but rather view their high school calculus course as a means of improving their performance in college calculus.

Procedure

The study was performed by matching students on three variables: I.Q., high school percentile rank, and college entrance examination board scores in mathematics. The matching was accomplished by using a three point span on I.Q., a five point span on class rank, and a twenty point span on college board scores. Students who had a year of calculus in high school were matched with students who had one of the other four year programs. (Those students with less than four years of math were not used in the study.) Of the 642 students who took calculus in this institution during the past four years, 228 (or 114 pairs) were used in the analysis. (114 students represent 56% of the entire population who had calculus in high school.) The difference between the means of the two groups was assessed using a t-test. Both the matching of students and the analysis of the data was done by computer. The t-test was performed using a "canned program" from the Statistical Package for the Social Sciences (McGraw-Hill Book Company, 1975). The null hypothesis was: There is no significant difference between the means of the two groups. The statistical results of the analysis are reported in Table 1 at the end of this report.

Results

The results of the study indicate that there is no significant difference between the means of the two groups in college calculus. Accordingly, there is no advantage to teaching calculus in high school. Notice also, that the difference of the means decreased from .062 in the first semester to .012 in the second semester. What little advantage the calculus group had, they seem to lose after one semester.

Unfortunately, the titles of the fourth year high school mathematics programs are not very descriptive, e.g., math analysis, math IV, etc. As a result we were not able to determine if any one program was particularly beneficial. Further research in this area is indicated. A study of the relationship between the training of the secondary mathematics teachers and the performance of their students may also prove enlightening. We would further recommend replication of this experiment in (a) different types of institutions, e.g., public and private universities, technical institutes, community colleges, and (b) different geographic areas of the country.

Recommendations

Since the Cambridge Conference, there have been many changes in the secondary curriculum which were made because of the direct influence of college faculty. It is time to evaluate these changes in terms of the purposes they were intended to serve. Recommendations for future secondary mathematics programs should be made in view of empirical evidence gained from present programs. This study indicates that we should consider other alternatives to calculus as an advanced math course in high school intended to prepare students for college.

Table 1

First Semester

Group	Mean	Standard Deviation	Standard Error	t-value	Degrees of Freedom	Probability
High Sch. Calculus	3.204	.939	.090	.53	214	.596
Non-calculus	3.142	.801	.078			

Second Semester

Group	Mean	Standard Deviation	Standard Error	t-value	Degrees of Freedom	Probability
High Sch. Calculus	3.255	.871	.084	.10	210	.921
Non-calculus	3.243	.795	.078			

AN INVESTIGATION OF THE RELATIONSHIP BETWEEN SELECTED PERSONALITY CHARACTERISTICS OF EIGHTH-GRADE STUDENTS AND THEIR ACHIEVEMENT IN MATHEMATICS

Daryle Cline May
Jacksonville University

Purpose

The primary purpose of this study was to investigate the relationship between the sensing and intuitive personality characteristics, as measured by the Myers-Briggs Type Indicator (MBTI), and mathematics achievement in computation, concepts, and applications at the eighth grade level. A secondary purpose was to investigate the relationship between these sensing and intuitive personality characteristics and mathematics attitudes and intelligence. Direction for the study came from the coordination of certain questions connected with mathematics and personality types. The following questions related to mathematics achievement, intelligence, and attitudes were considered: (1) Do eighth-grade students who are classified as sensing or intuitive MBTI personality types differ in mathematics achievement in computation, concepts, and applications? (2) Is there a difference in intelligence quotient between the students classified as sensing and those classified as intuitives? (3) Is there a difference in attitudes toward mathematics between the eighth-grade students classified as intuitive types and those classified as sensing types?

Procedure

The subjects chosen for this study were 295 eighth-grade students from a junior high school in Gainesville, Florida, during the academic school year 1970-71. The data for this study were obtained from the use of four instruments: (1) the Myers-Briggs Type Indicator; (2) the Stanford Achievement Test; (3) the Dutton Arithmetic Attitude Scale; and (4) the California Test for Mental Maturity. After completing the MBTI, each student was identified as a sensing or an intuitive personality type. Once the students were placed in either of the two groups, the sensing type students' scores on arithmetic computation, concepts, and applications; attitudes; and intelligence were compared with the scores made by the intuitive type students.

The data were analyzed by a two-tailed Student-t distribution with unpaired data and a chi square analysis to determine if the sensing and intuitive type students do show a statistically significant difference in scores on the tests listed above.

Results

The results of this study showed that there is a statistically significant difference in arithmetic achievement in computation, concepts, and

applications and in intelligence between the sensing and intuitive type students. The study also showed that there was no significant difference between attitudes toward mathematics of the two personality types.

Recommendations

This study suggests that the intuitive type students, as a group, performed better than the sensing type students on both the mathematics achievement test in computation, concepts, and applications, and the intelligence test. On the other hand, there appeared to be no statistically significant difference between the attitudes of the intuitive and sensing type students toward mathematics, as measured by the Dutton Arithmetic Attitude Scale.

Using the results of this investigation, and accepting the basic assumption that individuals have different learning patterns and personality characteristics, mathematics teachers, as well as other educators, should consider these factors when engaged in the process of educating children. To help the sensing type students improve their achievement in mathematics, as well as to provide a challenging atmosphere for the intuitives, teachers should consider instructional programs that are more suited to the nature of these differing personality types.

THE EFFECT OF THE LENGTH OF THE NUMBER LINE AND THE NUMBER OF LINE SEGMENTS IN EACH UNIT SEGMENT ON SEVENTH-GRADE STUDENTS' ABILITY TO LOCATE PROPER FRACTIONS ON THE NUMBER LINE

Carol P. Novillis
Florida International University

Purpose

Novillis (1973) reported that associating a point on a number line with a proper fraction was a great deal more difficult for intermediate grade students than associating a proper fraction with a part-whole model where the unit was a set. Payne (1975) also reported that elementary students had more difficulty with the number line model than with the area model (part-whole model).

The question posed based on these results is: "Why was the number line model so much harder than the other two types of models?" This investigator hypothesizes that the length of the number line might be part of the problem. It could be that when the number line is of length greater than one, some students disregard the scaling and treat the number line that is depicted as a unit regardless of its length. An important difference between a part-whole model and a number line model is that in the number line model the students need to also attend to the scaling. Hence a number line model implies a length greater than one. Whenever a number line of length one is used then the number line model is not being completely tested. For in this case the number line is really another part-whole model where the unit is one of length rather than area which is traditionally used.

One purpose of the study being proposed is to grade (1) students' ability to associate a proper fraction with a point on a number line when the number line is of length one and of length two; and (2) students' ability to associate a proper fraction whose denominator is n with a point on the number line, when the number of line segments into which each unit segment has been separated equals n and 2n. Also of interest is the establishment of the hierarchical relationships among the four types of number line subconcepts that occur when both length of number line (1 or 2) and number of line segments in (n or 2n) are considered.

Procedure

Locating Fractions on the Number Line, a test of sixteen items was constructed to measure the students' attainment of the four number line subconcepts. A student had to respond correctly to at least 75% of the items on a subtest in order to be described as attaining criterion on that subtest.

In a pilot study conducted in May 1975, eighty-one seventh-graders in Miami, Florida were administered the test, Locating Fractions on the Number Line. The results of analyzing the data using two ratios adapted by Novillis (1973) were that the following dependencies occurred:

1) associating a point on a number line of length one where the number of line segments in each unit segment was n (Subconcept A) was subordinate to

   a) associating a point on a number line of length two where the number of line segments in each unit segment was n (Subconcept B), and

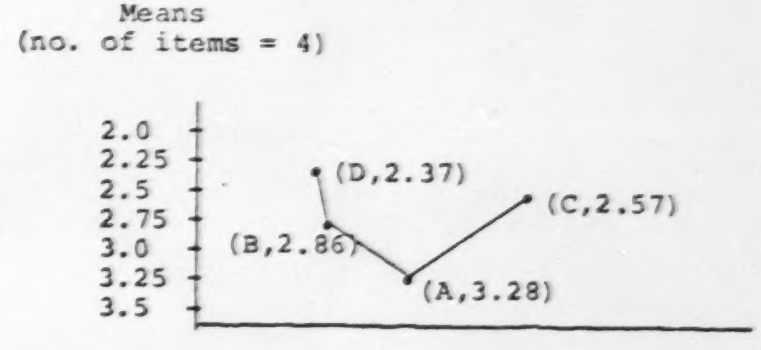
   b) associating a point on a number line of length one where the number of line segments in each unit segment was 2n (Subconcept C).

2) Subconcept A and Subconcept B were subordinate to associating a point on a number line of length two where the number of line segments in each unit segment was 2n (Subconcept D).

Table 1 illustrates the supported dependencies and the level of each subconcept relative to the others in the hierarchy based on the mean of the subtest that measured the behavior associated with it.

Table 1

Dependencies Supported With Subconcepts Ordered by Means
(n = 81)



In Fall 1975 the data will be collected for approximately 250 seventh-grade students in Miami.

References

Novillis, Carol F. An analysis of the fraction concept into a hierarchy of selected subconcepts and the testing of the hierarchical dependencies at grade levels 4, 5, and 6. Doctoral dissertation, The University of Texas at Austin, 1973.

Payne, Joseph. Directions for Research on Fractions. Paper presented at University of Georgia, March, 1975.

ASPECTS OF TEACHER DISCOURSE AND STUDENT LEARNING OF A MATHEMATICAL CONCEPT

Lyle R. Smith
Augusta College

James H. Rollins
Texas A & I University

Purpose

Empirical research has made little progress in determining the relative effectiveness of such global concepts as "democratic classrooms," "teacher indirectness," "teaching by discovery," or "lecturing." Perhaps the single most relevant suggestion for researchers of teacher effectiveness is that they focus upon manageable teacher skills that can be observed and quantified.

The purpose of this study was to examine discourse of mathematics instructors who teach the first year course in high school algebra and to determine if certain aspects of teacher discourse are related to student learning of mathematical concepts. Twenty teachers, who each had at least two years of experience teaching mathematics and who each taught in the Austin (Texas) Independent School District, participated in the study. All twenty teachers used the same algebra textbook (Dolciani and others, 1970). Each teacher taught a lesson over direct variation to one of his regularly scheduled algebra classes. Approximately one month before the lessons were taught, each teacher was given a list of lesson objectives that outlined the content over which the students would be tested. Immediately after each lesson was taught, a 14-item posttest which focused on the lesson objectives was administered. Two criterion measures of lesson effectiveness were obtained. One of these measures was the raw mean posttest scores for the classes, which involved the entire sample of 455 students. Of these 455 pupils, 350 had previously taken the California Achievement Test (CAT). Thus, for the sample of 350 students, raw class posttest means were adjusted by using the CAT scores and an analysis of covariance. The adjusted class means were used as the second criterion measure of lesson effectiveness.

Twelve variables pertaining to teacher discourse were investigated. Two of these variables were quantified by timing portions of the tape recorded lessons. These variables are total lesson time and percent teacher talk. Lesson times ranged from a low of 12 minutes 18 seconds to a high of 31 minutes 39 seconds. Teacher talk ranged from 55.8 percent to 94.0 percent. The other ten variables were quantified after coders were trained to identify them by studying transcripts of the tape recorded lessons. The coders were trained to adequate intercoder reliability in groups of three. The ten variables are discussed below.

A. Vagueness Terms: Hiller, Fisher, and Kaess (1969, p. 670) referred to vagueness as ". . . a psychological construct which refers to the state of mind of a performer who does not sufficiently command the facts

or the understanding required for maximally effective communication." In their study of classroom lecturing in high school social studies, Hiller, Fisher, and Kaess (1969) defined categories of vagueness terms and found that the frequency of vagueness terms used by the instructor was negatively correlated with lecture effectiveness. The investigator attempted to verify this finding by computing the average number of vagueness terms per minute of teacher talk for each lesson. The average number of vagueness terms per minute of teacher talk ranged from 1.05 to 3.97. The present investigation deviates from the 1969 study in that Hiller, Fisher, and Kaess defined "OK" as a vagueness term, whereas "OK" was treated as a separate variable in the present study since it occurred so frequently in the lessons.

B. Teacher Responses of "OK": Hiller, Fisher, and Kaess (1969) classified the teacher response "OK" as a vagueness term, the rationale being that "OK" appeared to be a term which was nervously repeated by the teacher and lent nothing to the substantive content of the lesson. However, Hiller, Fisher, and Kaess (1969, p. 671) reported that "OK" occurred less frequently in the lessons that 17 of the other phrases which they classified as vagueness terms, whereas, in the present study, "OK" occurred more frequently in the lessons than any of the other phrases classified as vagueness terms. The average number of "OK's" per minute of teacher talk ranged from 0.00 to 4.92.

C. Mazes: Hunt (1968, p. 10) referred to a maze as any unit of discourse that does not make semantic sense. In the present study, false starts or halts in speech, redundantly repeated words, or simply tangles of words were classified as mazes. The average number of mazes per minute of teacher talk ranged from 0.86 to 6.09.

D. Student Verbal Participation: Flanders (1970), Coats (1966), and Soar (1968) reported that degrees of pupil verbal participation positively influenced pupil achievement. In the present study, two variables pertaining to student verbal participation were studied. One variable, the number of teacher initiated student responses, quantified the number of times students responded after the teacher solicited a response. the other variable, the number of pupil initiated student responses, quantified the number of student responses which were not solicited by the teacher. Teacher initiated responses ranged from 4.0 per lesson to 153.0 per lesson. Pupil initiated responses ranged from 1.0 per lesson to 62.0 per lesson.

E. Lesson Objectives: Approximately one month before the teachers taught their lessons, they were given a list of objectives for the lessons. These objectives focused on the concept of direct variation. The purpose of this variable was to quantify the number of lesson objectives the teachers attempted to meet, to examine the degree to which the objectives were dealt with according to hierarchical sequences, and to determine whether the degree to which lesson objectives were focused upon and the sequences in which they were dealt with were correlated with posttest achievement. The points awarded teachers for dealing with lesson objectives ranged from 2.0 to 10.0.

F. Examples and Applications: Coders were trained to identify examples and applications in the lessons. No attempt was made to distinguish examples from applications. Those examples and applications which focused upon the lesson objectives are referred to as "relevant examples and applications." Those examples and applications which did not focus upon the lesson objectives are referred to as "irrelevant examples and applications." Thus, a total of four variables pertaining to examples and applications were examined. These variables are: (1) the total frequency of examples and applications, (2) the frequency of relevant examples and applications, (3) the percent of relevant examples and applications, and (4) the frequency of irrelevant examples and applications. The total number of examples and applications per lesson ranged from 14.0 to 52.0.

Results

The product moment correlation coefficients were computed between each of the twelve independent variables and the two criterion measures. The findings revealed that the variable pertaining to lesson objectives was correlated positively with posttest achievement ($p < .02$), as were the percentage of relevant examples ($p < .01$), the frequency of relevant examples ($p < .05$), and the frequency of "OK's" in teacher talk ($p < .05$). The frequency of vagueness terms in teacher talk was correlated negatively with posttest achievement ($p < .10$), as were the frequency of irrelevant examples ($p < .10$) and the percent teacher talk ($p < .20$). These results are summarized in the following table.

Correlation Coefficients for Independent Variables with Criterion Measures

Variable	Correlation with Cov.-Adj. Means	Significance* Level	Correlation with Raw Means	Significance* Level
1. Percent Relevant Examples	.6521	.002	.5913	.01
2. Lesson Objectives	.5835	.01	.5403	.02
3. Relevant Examples	.4963	.05	.4944	.05
4. Ave. "OK's" Per Minute Teacher Talk	.4440	.05	.4578	.05
5. Irrelevant Examples	—.5156	.02	—.4208	.10

Table (continued)

6. Average Vagueness Terms Per Min. Teacher Talk	—.4054	.10	—.4893	.05
7. Percent Teacher Talk	—.3341	.20	—.3687	.20
8. Total Lesson Time	.3043	.20	.2316	n.s.
9. Total Examples and Applications Per Lesson	.2252	n.s.	.2952	.30
10. Ave. Mazes Per Min. Teacher Talk	—.2334	n.s.	—.2753	.30
11. Total Teacher Initiated Responses	.1385	n.s.	.2430	n.s.
12. Total Pupil Initiated Responses	—.0424	n.s.	—.0456	n.s.

* Two tailed tests

Product moment correlation coefficients were also computed between each independent variable and the other independent variables. Further hunches pertaining to teacher effectiveness can be supported by studying these coefficients.

To conclude, the results of the study indicate that it would be appropriate to train teachers to list explicitly the objectives of their lessons and to carefully plan examples which focus on these objectives. Evaluation of teachers should be based on specific criteria, such as the number of lesson objectives met and the frequency of relevant examples.

References

Coats, E. D. Investigations and simulation of the relationships among selected classroom variables. Unpublished doctoral dissertation, The University of Michigan, 1966.

Dolciani, M. P.; Wooten, W.; Beckenbach, E. F.; Jurgensen, R. C.; and Donnelly, A. J. Modern School Mathematics: Algebra I. Boston: Houghton Mifflin Company, 1970.

Flanders, N. A. Analyzing Teaching Behavior. Reading, Massachusetts: Addison-Wesley Publishing Company, 1970.

Hiller, J. H.; Fisher, G. A.; and Kaess, W. A computer investigation of verbal characteristics of effective classroom lecturing. American Educational Research Journal, 1969, 4, 661-675.

Hunt, K. W. An instrument to measure syntactic maturity. ERIC, No. ED 020 926, 1968.

Soar, R. S. Optimum teacher-pupil interaction for pupil growth. Educational Leadership, 1968, 26, 275-280.

RESEARCH REPORTING SECTION #7

Presider: LARRY SOWDER, Northern Illinois University
DeKalb, Illinois

Reports: COGNITIVE STYLE AND MATHEMATICS LEARNING: THE INTERACTION OF FIELD INDEPENDENCE AND INSTRUCTIONAL TREATMENT IN NUMERATION SYSTEMS

DOUGLAS B. MCLEOD, San Diego State University
San Diego, California

THOMAS P. CARPENTER, University of Wisconsin-Madison
Madison, Wisconsin

ROMUALDAS SKVARCIUS, Boston University
Boston, Massachusetts

EFFECTS OF STUDENT AND TEACHER COGNITIVE PREFERENCES ON JUNIOR HIGH MATHEMATICS ACHIEVEMENT

WILLIAM J. O'DONNELL, Denver Public Schools
Denver, Colorado

RELATIONSHIPS BETWEEN VARIABLES IN LEARNING AND MODES OF PRESENTING MATHEMATICS CONCEPTS

MARY K. ABKEMEIER, University of Pittsburgh
Pittsburgh, Pennsylvania

FRED H. BELL, University of Pittsburgh
Pittsburgh, Pennsylvania

A COMPARISON OF TWO APPROACHES TO THE ASSESSMENT OF DEDUCTIVE REASONING ABILITIES

LARS C. JANSSON, University of Manitoba
Winnipeg, Manitoba, Canada

COGNITIVE STYLE AND MATHEMATICS LEARNING: THE INTERACTION OF FIELD INDEPENDENCE AND INSTRUCTIONAL TREATMENT IN NUMERATION SYSTEMS

Thomas P. Carpenter
University of Wisconsin-Madison

Douglas B. McLeod
San Diego State University

Romualdas Skvarcius
Boston University

Purpose

Research on the Aptitude Treatment Interaction (ATI) hypothesis of Cronbach has generally been inconclusive. One difficulty in much of this research has been the identification of aptitudes that are both based in psychological theory and related to instruction in mathematics. Recent research reported by Witkin indicates that his field-dependence-independence dimension of cognitive style may be a useful aptitude measure for ATI research in mathematics education. (See, for example, Research Bulletin 73-11 of the Educational Testing Service, The Role of Cognitive Style in Academic Performance and in Teacher-student Relations, by H. A. Witkin.)

Field-dependence-independence is an example of a cognitive style that is related to the ability to separate part of a field from the whole. While this ability to disembed an element from context is assessed through a perceptual task, it is also related to certain kinds of intellectual performance and social behavior.

There is substantial evidence that field independence is correlated with intellectual performance in mathematics. Specifically, it is related to the ability to identify the salient elements of a mathematical problem. Recent studies also indicate that differences in this dimension may be associated with an individual's capacity to function in different instructional environments. Field independence seems to be positively correlated with an individual's reliance on inner standards as opposed to external direction. Thus, it may be hypothesized that level of guidance is related to cognitive style; field-dependent students seem to respond better when there is more explanation from the teacher, while field-independent students are likely to be more adept at working independently and making discoveries without much assistance. A major purpose of the study was to test this hypothesis.

A second purpose of this study was to investigate the relationship between field-dependence-independence and level of abstraction. Since field independence is related to the ability to identify the significant attributes of mathematical concepts, a level-of-abstraction dimension that varies the representation of the concepts was included in the study. This dimension is also appropriate since earlier research indicated that level of abstraction may interact with level of guidance; both of these dimensions should be accounted for in studies related to discovery learning.

In summary, this study investigated the relationship of the field-dependence-independence dimension of cognitive style to instructional treatments based on two levels of abstraction crossed with two levels of guidance.

Procedure

Four parallel instructional treatments were prepared on the topic of addition and subtraction of whole numbers in bases other than ten. The treatments varied only in level of guidance and level of abstraction: minimum guidance with manipulative materials (Min-M), maximum guidance with manipulatives (Max-M), minimum guidance with only a symbolic presentation (Min-S), and maximum guidance with a symbolic presentation (Max-S). The manipulative materials used were multibase arithmetic blocks.

The 116 subjects for the study came from four sections of a mathematics class for prospective elementary teachers. All subjects were pretested over prerequisite concepts of place-value numeration and randomly assigned to one of the four treatment groups.

Each student worked individually on a packet of printed materials designed for his treatment group. While the printed materials varied in level of guidance and level of abstraction, they covered the same content (addition and subtraction in bases other than ten), using the same sequence and the same problems, and they provided the same amount of practice.

Minimal guidance was provided for the Min-M and Min-S groups; their printed materials consisted mostly of questions and problems. If a student asked the instructor for help, he was referred back to the printed materials or asked to solve a simpler problem. Students in the Min-M treatment were expected to make discoveries using the blocks, while the Min-S group was often asked to reason by analogy, using their knowledge of base ten numeration. On the other hand, the materials for the Max-M and Max-S groups presented the concepts in complete detail. An example of each type of problem was worked out and explained, and if a student asked a question, the instructor answered it directly and completely. Students were given an hour to complete the treatment.

The posttest consisted of two parts. In the first subtest (QS), all problems were presented symbolically and no blocks were available. In the second subtest (QM), students were encouraged to use multibase blocks, and certain problems were presented as pictures of blocks. QS and QM were parallel tests covering mainly addition and subtraction in bases other than ten. Transfer items on multiplication and division were also included. Four weeks later QS and QM were readministered as a retention test. The NLSMA version of the Hidden Figures Test was administered at the same time to provide a measure of field-dependence-independence.

Results

Results are currently being analyzed using ANCOVA to test for differences between treatment means and heterogeneity of regression to test the ATI hypothesis. Preliminary analysis of the data indicates that there are probably significant level-of-abstraction main effects on subtest QM, but not on QS. There appear to be no significant main effects due to level of guidance.

Preliminary analysis also indicates that there is no interaction between field-dependence-independence and level of abstraction. However, there does appear to be a disordinal interaction between the aptitude measure and level of guidance. This apparent difference in the performance of field-dependent and field-independent students is in the direction predicted by Witkin's theory of cognitive style.

EFFECTS OF STUDENT AND TEACHER COGNITIVE PREFERENCES ON JUNIOR HIGH MATHEMATICS ACHIEVEMENT

William J. O'Donnell
Denver Public Schools

Purpose

The concept of the "Cognitive Preference Test" originated in the early 1960's in response to widespread curriculum reform efforts. Cognitive preferences were defined by Heath (1964) to be the "differing modes of attending to the subject matter of a course" (p. 241). The test which he developed was called the Cognitive Preference Test: High School Physics (CTP:HSP). Heath explained his test in this way:

> "The interest is not in whether the student can identify correct or incorrect information but rather in what he is likely to do with information intellectually (p. 241)."

Using the CPT:HSP, Heath found that a traditional physics course and the PSSC physics course produced different cognitive preferences in two groups of students. Subsequent research by Payette (1967), Barnett (1972), and Kempa and Dubé (1973) found that a significant relationship existed between the cognitive preferences of students and achievement in science courses. Travers, Heath and Caben (1967) developed a cognitive preference test for use in mathematics courses, but to this date no research has been reported in the literature which pertains to mathematical cognitive preference.

This study was designed to investigate the effect of teacher and student cognitive preferences on student achievement as well as the interaction effect of teacher and student cognitive preferences on achievement.

Procedure

To test the research hypotheses, the following instruments were used: Cognitive Preference Test: Mathematics (CPTM) and a shortened version of the Mathematics: Concepts and the Mathematics: Computation subtests of the Science Research Associates Achievement Series, Form E, Red Level.

The CPTM consists of 32 items; each item containing an option for graphic, verbal, and symbolic representation of mathematical information. On each item, the subject is to choose the mode of representing mathematical information that he or she most prefers. The test yields a score for each of the three preference categories.

The CPTM was administered to 33 junior high teachers during the summer and fall of 1974. From this group seven teachers were classified as being "graphic" teachers while seven teachers were classified as being "symbolic" teachers. Only four teachers could be classified as being "verbal" teachers.

With so few teachers in the verbal category, the investigator could not reasonably assume that the teacher effects in this category were randomly distributed. Therefore, the mode for graphic representation was omitted for the teachers in this study.

Each teacher who participated in the study administered the CPTM and the achievement test to his or her students during the first week of December 1974. The students were classified by cognitive preference on the basis of their scores on the CPTM. The achievement test was readministered to the students during the first week of May 1975.

Analysis

The data was analyzed with a 2 x 3 analysis of covariance (ANCOVA) model. The independent variables in the experimental design were the teacher's cognitive preference and the student's cognitive preference. The dependent variable was the student's score on the mathematics achievement test. The student's score on the pretest was used as a covariate. The ANCOVA was also performed on the raw concept and computation scores.

Results

There were nine null hypotheses tested for significance at the .05 level. The results of the analysis of covariance are summarized as follows:

1. There is no significant interaction effect between teacher and student cognitive preferences on the student's mathematical achievement.

2. There is no significant effect of student cognitive preference on student mathematical achievement.

3. There is no significant effect of teacher cognitive preference on the student's mathematical achievement.

4. There is no significant interaction effect between teacher and student cognitive preferences on the student's mean computation score. Aside from the student's pretest score on the computation subtest, the interaction of teacher and student cognitive preferences proved to account for the most variance in the student's posttest score.

5. There is no significant effect of student cognitive preference on student computation achievement.

6. There is no significant effect of teacher cognitive preference on the student's computation achievement.

7. There is no significant interaction effect between teacher and student cognitive preferences on the student's mathematical concept achievement.

8. There is no significant effect of the student's cognitive preference on student concept achievement.

9. At the .05 level there is no significant effect of teacher cognitive preference on student concept achievement. The effect is significant at the .10 level.

Conclusions

The findings of this study justify the following conclusions:

1. The interaction between teacher and student cognitive preferences does not have a significant effect on either computation or concept achievement or total mathematics achievement in seventh and eighth grade mathematics.

2. The cognitive preference of the student has no significant effect on either computation or concept achievement or total mathematics achievement in seventh or eighth grade mathematics. This finding is in contradiction to the research findings in other disciplines.

3. The cognitive preference of the teacher has no significant effect on the computation achievement or total mathematics achievement of seventh and eighth grade students. The teacher's cognitive preference may have an effect on concept achievement, though, with the students of "symbolic" teachers achieving slightly higher.

References

Barnett, H. C. An investigation of relationships among achievement, perception of teacher style, and cognitive preferences of students in tenth grade biology. Unpublished doctoral dissertation, University of Kentucky, 1972.

Heath, R. W. Curriculum, cognition, and educational measurement. Educational and Psychological Measurement, 1964, 24(2), 239-253.

Kempa, R. K. and Dubé, G. E. Cognitive preference orientations in students of chemistry. British Journal of Educational Psychology, November 1973, 43, 279-288.

Payette, R. F. Development and analysis of a cognitive preference test in the social sciences. Unpublished doctoral dissertation, University of Illinois, 1967.

Travers, K. S.; Heath, R. W.; and Cahen, L. S. Preferences for modes of expression in mathematics. Stanford Center for Research and Development in Teaching, Research Memorandum 7, Unpublished manuscript, Stanford University, 1967.

RELATIONSHIPS BETWEEN VARIABLES IN LEARNING AND MODES OF PRESENTING MATHEMATICS CONCEPTS

Mary K. Abkemeier
University of Pittsburgh

Fred H. Bell
University of Pittsburgh

Purpose

The purpose of this aptitude X treatment interaction research was to investigate relationships between figural and symbolic aptitudes and figural and symbolic modes of presenting mathematical concepts to secondary school students. In designing the study, the investigators classified students according to the two variables of sex and instructional modality preference in mathematics producing five experimental groups: (1) the total group, (2) the male group, (3) the female group, (4) the group preferring the figural instructional mode, and (5) the group preferring the symbolic instructional mode.

Specifically, the authors sought answers to the following questions:

Question 1. Does either a figural or symbolic mode of presenting mathematics concepts produce superior achievement in any of these five experimental groups?

Question 2. Do interactions exist between instructional mode and sex? between instructional mode used and instructional mode preferred?

Question 3. For each of the five experimental groups, what are the relationships between instructional mode and the set (or subsets) of 11 aptitudes (five figural, five symbolic, and one semantic) selected from Guilford's Structure-of-Intellect model?

Procedure

The mathematics concept of function and related concepts were incorporated into two printed programmed instructional sequences presented to 160 high school students. One program employed a symbolic mode of presentation and the other a figural mode. The "neutral" verbal material was the same in both programs. Information analysis of 12 randomly selected paired passages from the two programs showed that they contained the same mathematical content. Subjects were randomly assigned to study the material using either the figural or symbolic instructional programs for three class periods on consecutive days.

The following 11 aptitudes from Guilford's Structure-of-Intellect cube were selected as independent variables:

Cognition of Figural Classes	CFC
Cognition of Figural Systems	CFS
Convergent Production of Figural Transformations	NFT
Convergent Production of Figural Implications	NFI
Divergent Production of Figural Systems	DFS
Cognition of Symbolic Classes	CSC
Cognition of Symbolic Systems	CSS
Convergent Production of Symbolic Transformations	NST
Convergent Production of Symbolic Implications	NSI
Divergent Production of Symbolic Systems	DSS
Cognition of Semantic Systems	CMS

Two achievement tests, a Learning Test and Retention Test, were given to each student following study of the programmed materials. Students' total scores for each test were subdivided into three categories: scores for questions presented in a figural mode, for questions presented in a symbolic mode, and for questions presented in the "neutral" verbal mode. This division, according to question presentation mode on each test, was done to check for relationships between treatment mode and questioning mode.

Since the printed instructional programs were relatively abstract, the Shipley Abstraction Test was given to all potential subjects for this study. The 160 students for whom data were analyzed had abstraction ages of at least 14.2 years.

The conduct of the experiment, testing and instruction, was divided into four chronological stages:

Stage 1. Each student took the Shipley Abstraction Test, the Mathematics Modality Preference Test (an author-constructed test measuring a student's preference for either the figural or symbolic instructional mode), and answered a questionnaire to determine if he or she had prior knowledge of the function concept.

Stage 2. Each student read one of the figural or symbolic instructional programs in algebra class for three consecutive days.

Stage 3. Each student took the Learning Test during the mathematics class immediately following the reading of the instructional programs. Each student took the Retention Test one week later.

Stage 4. Between administrations of the Learning and Retention Tests, each student took 11 tests to measure Structure-of-Intellect aptitudes.

Results

A two-tail t-test (significance level 0.05) was used to answer Question 1. For the total group, significant differences between means of the figural and symbolic subgroups were found on the verbal and figural parts of the Learning Test. In the male group significant differences were found between means on the verbal and figural parts of the Learning and Retention Tests. All differences favored the figural treatment.

A fixed effects, two-way ANOVA for unequal n's with $p \leq 0.05$ was used to answer Question 2. No significant interactions between instructional material used and sex, or instructional material used and instructional material preferred were indicated. The analysis did show that females scored significantly higher than males on all eight dependent measures.

Multiple and simple linear regression analyses were used to answer Question 3. Multiple linear regression equations giving each dependent measure as a function of the 11 aptitude measures in the figural and symbolic subgroups of each experimental group, and simple linear regression equations giving each dependent measure as a function of each aptitude measure were computed. Also, an attempt was made, using stepwise elimination, to find a subset of the 11 aptitude measures which significantly accounted for a substantial proportion of the variance in each dependent measure, such that each aptitude measure in the subset made a significant contribution to the regression equation.

Statistically significant relationships were found between instructional mode and Structure-of-Intellect aptitudes, which were attributed primarily to the aptitude DFS. DFS was the best single predictor of achievement in both treatment subgroups of all five experimental groups. Achievement was positively correlated with DFS in all figural subgroups; and negatively correlated in all symbolic subgroups. Also, there was a statistically significant relationship between DFS and that part of achievement not explained by the other 10 aptitudes in both treatment subgroups.

The second best single predictor of achievement in the symbolic subgroups was CSS. Success was positively and significantly related to CSS in the symbolic subgroups of all five experimental groups. No relationships were observed in the figural subgroups.

The semantic aptitude CMS was significantly related to immediate and delayed achievement in the total figural group. The strength of the relationship decreased from the measure of immediate to delayed learning. The measure for CMS was the second most useful predictor of achievement (DFS was first) in the total and male figural subgroups.

Recommendations

To predict learning of mathematical content similar to content of this study by figural and symbolic presentations researchers should probably include DFS in their predictive set of aptitudes.

It appears that different verbal aptitudes are needed to learn from figural and symbolic materials.

The students' information processing mode may be a crucial factor, and should be considered in future aptitude X treatment interaction studies.

A COMPARISON OF TWO APPROACHES TO THE ASSESSMENT OF DEDUCTIVE REASONING ABILITIES

Lars C. Jansson
University of Manitoba

Purpose

Great strides have been made in the last decade in the assessment of logical thinking abilities. A number of studies carried out at elementary school through college level (e.g., Ennis, 1965, 1969; O'Brien, 1972; Jansson, 1975) have given us much information about children's abilities to handle various principles of deductive reasoning in both class and conditional forms. While a variety of assessment procedures have been used, most of these investigations have employed a pencil and paper test of the sort typified by that of the Cornell Critical Thinking Project (Ennis, 1965). A major concern with this form of assessment is the question of generality. To what extent does it truly measure what subjects are capable of in varying situations which call for deductive reasoning.

The purpose of the present study is to compare results of studies using written tests of the type described above with those found by a different sort of assessment procedure. Deductive reasoning here is confined to simple conditional arguments where the first premise is of the form $p \rightarrow q$. The other measure of deductive reasoning ability was the Four Card problem, a series of tasks built around a display of four cards and a rule in the form of an "if. . .then" statement. More specifically, answers were sought to the following questions:

1. Do adolescents differ in their measured ability to judge written verbal deductive arguments and in their ability to apply deductive reasoning principles in the Four Card tasks?

2. Do differences in mastery level on these two tests vary by principle of reasoning?

3. Are there patterns of reasoning, other than the conventional logic, which are used consistently by adolescents?

4. To what extent do findings of other investigations hold for yet another sample?

Procedure

The experimental sample was drawn from grade 8, 10, and 12 mathematics students in four Winnipeg area schools, a total of 464 subjects. All subjects were randomly assigned to take one of two forms of a pencil and paper test with items of the following type:

Suppose you know that

Premise

Premise

Then would this be true?

Conclusion

Three responses are possible and these were defined for the subjects as follows:

1. YES	It must be true.
2. NO	It can't be true.
3. MAYBE	It may be true and it may not be true. You weren't told enough to be certain whether it is "YES" or "NO".

Each test consisted of 32 items, 8 for each of four principles of reasoning: modus ponens, inversion, conversion, contraposition. Furthermore, the two tests varied in content and both tests contained a systematic variation in the location of negations within the arguments. In addition to the written test, each subject was given the Four Card tasks presented with an overhead projector and a specially constructed answer sheet.

In order to do a comparison of the type desired, mastery criteria were defined for each of the two types of tests, then proportions of students mastering a principle on each of the two assessments for a grade level were compared using a test of the significance of the difference between two proportions for correlated samples. Mastery on the written test was defined to be 6 out of 8 items correct for a principle. Quantifying the results of the Four Card tasks in a useful manner was somewhat more difficult. In this investigation it was decided to arbitrarily define a minimum performance for each principle on each task that would be acceptable as not violating an understanding of that principle. Each such criterion was defined as a conjunction of specified performances on the three Four Card tasks.

Results

Analyses were made for each grade level by principle as described above. Performance on the two types of tasks did not differ significantly for the invalid principles of inversion and conversion at any of the three grade levels. In fact, the mastery rate on both was extremely low. On the other hand, for the contrapositive, differences in performance were clearly significant, with the written test being easier at all grade levels. The results for the modus ponens were mixed. Differences were significant at grades 8 and 10, but had disappeared by grade 12.

Further analyses of actual responses to the Four Card problem were made. It appears that much of the difficulty that students encounter is

through assuming the validity of the converse, i.e., given $p \rightarrow q$, it is assumed also that $q \rightarrow p$. O'Brien (1972) has suggested that $p \rightarrow q$ is interpreted as $(p \cdot q) \vee (p \cdot \bar{q})$ rather than as the correct $(p \cdot q) \vee (p \cdot \bar{q}) \vee (\bar{p} \cdot \bar{q})$. The present analysis suggests that while as many as 20% of the subjects may make this interpretation, it is not really wide spread. Further analyses of interactions and recognizable patterns in responses will be reported.

References

Ennis, R. H. and Paulus, D. H. Critical Thinking Readiness in Grades 1-12. (Phase 1: Deductive Reasoning in Adolescence). Ithaca, New York: The Cornell Critical Thinking Project (ERIC No. ED 033 818), 1965.

Ennis, R. H., et. al. Conditional Logic and Children (Phase 11C). Ithaca, New York: The Cornell Critical Thinking Project (ERIC No. ED 040 437), 1969.

Jansson, L. C. The judgment of simple deductive arguments by pre-service elementary school teachers. Alberta Journal of Educational Research, 1975, 21(1), 1-10.

O'Brien, T. C. Logical thinking in adolescents. Educational Studies in Mathematics, 1972, 4, 401-428.